VIII. *On the Constant of Refraction determined by Observations with the Mural Circle of the Armagh Observatory.* *By the* Rev. T. R. Robinson, D. D., *Member of the Royal Irish Academy, and other Philosophical Societies.*

Read 11th January, 1841.

IT may, perhaps, appear presumptuous in me to approach a subject which has already occupied so many of the greatest masters of mathematical science, and in the opinion of many is exhausted. But if we look without prejudice at the labours of Laplace, Bessel, Ivory, and Plana, besides many others of less renown, and carry our examination a little beyond the mere analytical work, we shall find that the problem of astronomical refraction has not been rigorously solved by theory, and I am even inclined to think never can be. All it appears to me that theory can be expected to perform, is the supplying astronomers with ready means of approximating to tables of refraction, which shall satisfy their observations; and on the other hand they are bound to remember, that such tables, however carefully verified for one observatory, may be defective when tried at another.

For in fact it is universally assumed in these investigations, that the atmosphere is arranged, with the surfaces of equal density spherical and concentric to the earth; this gives the differential of refraction in function of the density and distance from the centre. Now, firstly, this fundamental hypothesis is not even approximately true. Near the earth, the surfaces of equal temperature (and therefore of equal density) must depend on the figure of the ground; the air over a hill must be very differently circumstanced in respect of heat, from that at the same height over a deep valley. Forests, large bodies of water, and the vicinity of cities must exert a similar disturbing influence, and that to an extent which cannot be neglected. In a set of hourly observations, made some years since on the altitude of my meridian mark, I found an increase of refraction,

amounting sometimes to 13″, when the intervening valley was overshadowed by clouds, though the meteorological indications at the observatory remained the same. But how much greater would the disturbance of a star have been whose light must have passed through many miles subject to these anomalies? For we have no reason to suppose that they are confined to the immediate vicinity of the earth's surface; they must extend as far as the clouds, (whose existence shews an irregular distribution of heat,) or at least six miles high; more than three times the height of Quito, at which Bouguer found the refraction only two-thirds of what it is at the level of the sea. Some remarkable facts respecting the variation of terrestrial refraction, when the ground is covered with snow, and immediately after sunset, are given by Struve, in his Gradmessung, but one still more in point is mentioned by the Rev. G. Fisher,* in the Appendix to Parry's Second Voyage, page 175. He found, while observing at Igloolik, that at temperatures of from 20° to 30° below Zero, and at an altitude of 3° 8′, the refractions of Sirius were about a minute less when observed over open sea to the south-east, than over land covered with snow or ice, to the south-west. The existence of these local anomalies can only be ascertained by low refractions; and therefore theory is in such cases unavailing.

But secondly, even were the hypothesis on which the differential equation of refraction is based strictly true, yet that equation cannot be integrated without assuming a relation between its variables, their real relation being unknown. Philosophers have been guided in this, either by supposed conformity to the law of nature, or by facilities of integration; but in both cases their results cannot be supposed to have any value except as far as they are confirmed by observation, and therefore all must be pronounced alike empirical. But at low altitudes observations are both difficult and uncertain, and therefore it is by no means easy to pronounce on the results of a given hypothesis; so that besides that lately published by Biot (but which I believe has not yet been applied to construct refraction tables) there are at least four of high authority; that of Newton, as modified by Bessel, supposing the temperature uniform, but changing the modulus of atmospheric elasticity by an experimental co-efficient; that of Simpson,

* To whom I am indebted for much valuable information respecting the important observations published there, and indeed for my acquaintance with the book itself.

assuming the density to decrease uniformly as the height increases; that of Laplace, expressing the density by a product of two factors, representing the preceding hypotheses, and that of Ivory, supposing it as $\left(1-\frac{r-a}{5\,l}\right)^4$.* Now these are obviously mere arbitrary assumptions, and the verifications which some of them are supposed to receive by exhibiting the decrease of temperature at a small elevation, and the barometric formula for heights, are worth little; the first being unknown at any given place,† and the second being a consequence of any law which will make the temperature decrease nearly uniformly within a few thousand feet. The slightest attention to meteorological facts will show that there cannot be any general formula expressing the density in terms of *the height alone*, and that even could it be found for one place by experiment, it would be entirely inapplicable to any other. It is certain, that between the tropics there is an ascending current of heated air, replaced by a stream of cooler from the north, while it flows towards the poles, descending in its turn and giving out its heat; and it is therefore equally certain that the law of atmospheric temperature must depend on the latitude. It is not impossible, that in the arctic regions we may find a uniform temperature, or even an increase on ascending. Such must indeed be the case, if there be any truth in the conclusions of Fourier, or Poisson, respecting the temperature at the termination of our atmosphere; for if with the former we suppose it $= -58°$ of Fahrenheit, or with the latter, much more elevated, approaching 32°, yet cold below either has been observed by northern travellers. At a given place we might, perhaps, by aeronautic investigations, ascertain the law of decreasing density and temperature, for a certain epoch; but it is highly probable, that this would not obtain when the sun had a different declination, or the weather was different;‡ it is unquestionable, that it would be

* The last appears the best, but it is to be regretted that Mr. Ivory has assumed the use of the internal thermometer, and not given separate reductions for the temperature of the barometer. This last also applies to the very convenient tables of Bessel's Refractions, given by Mr. Airy.

† Because the decrease in free air cannot be the same as that observed on the side of a mountain, and in contact with a mass of matter influenced both by the air and the earth's internal heat.

‡ In the celebrated ascent of Gay Lussac, the temperature at Paris was 87° Fahrenheit, so that the air cannot have been in a normal condition: the meteorological instruments below should have been noted every few minutes, and the times of observation above given. In the published

disturbed by wind, or variations in the hygrometric state of the air. And it must be remembered, that at least three-fourths of the entire refraction are produced in the region which is thus affected; and that in observation we find differences of 15 or 20 seconds in the same star, when the thermometer, barometer, and hygrometer of the observatory shew no change.

It appears to me, therefore, vain to expect an *a priori* solution of the problem of astronomical refraction, and that it will always be necessary to reform by observation whatever tables may be proposed to us. The tables of Bessel or Ivory—(if the refractive and thermometrical constants of the latter were corrected, I should prefer them)—are sufficiently exact for this purpose in the observatories of Europe.* Down to 74° zenith distance, it is known, that the law of density has no sensible effect on the refraction; and in ordinary cases this is sufficient for the astronomer, who seldom observes so near the horizon, because there the fluctuations of a star are so great, that a great number of observations are necessary to give even moderate precision. But he *must* occasionally observe, under such circumstances, comets and planets; and, besides, it is necessary for an accurate determination of the principal constant, that he should go as far from the zenith as is possible, without risking the certainty of his correction. In my latitude, at 74° zen. distance, an error in the constant is only doubled; and the average discordance of observation will be near a second; so that were we limited to the use of stars above this altitude, it would be almost

account it is stated, that the thermometer was steady at 30·75 cent. As light clouds existed far above the balloon there must have been an evolution of heat from their formation. Still it is to be wished that the experiment were repeated.

* In the Arctic regions all the tables fail completely. I give a couple of instances from the Appendix to Parry, already noticed, p. 209. They are Nos. 25 and 29. The first gives from 108 observations, the refraction = 665″.9 at zen. dist. 84°.13′, 82, Bar. 29.79, A. T. + 45, Ext. T. − 35°.9. After correcting for latitude, Bessel's refraction is 18″.72 in defect, Ivory's 13″.27, and mine 20″.71. Again, 32 observations give refraction = 342″.5 at 79°40′. 61, bar. 29.86, A. T. + 45°, E. T. − 26°.7. Here Bessel's is 40″.31 in excess, Ivory 31″.66, and mine 22″.78. It seems to follow from these and similar instances, that in such extreme cases the arrangement of the atmosphere must be regulated by very different laws from those that prevail in more temperate latitudes; and it seems equally obvious, that its influence on refraction commences much nearer the zenith. It is my intention to recur to these Arctic observations in a subsequent communication on the lower refractions.

impossible to determine it to the tenth of a second. But it is practicable to go about 10° lower, by a principle, first, I believe, remarked by Laplace; namely, that the refraction computed on the hypothesis of uniform temperature is greater than the truth, and on the hypothesis of uniformly decreasing density less, and that the mean of the two is nearly exact. For instance, Laplace gives for the horizontal refraction, (T = 32°; barometer, 29.92,)

U. Temp. . . .	2394″.6	288.6
Observed . . .	2106 .0	
Uniform decrease of dens.	1824 .1	281.9

The arithmetical mean = 2109.3; the geometrical = 2090. Ivory finds (T = 50, bar. = 30.00,)

U. T.	2254.5	223.0
French tables . . .	2031.5	
U. D. D.* . . .	1722.7	308.8

In this case the second deviates the most, arith. mean = 1988.6; geometrical = 1970.7.

At zen. dist. 85° 16′.70, T = 54.2, bar. 30.24, I find with Ivory's constant,

U. T.	624.3	3.7
Ivory's first tables . .	620.6	
U. D. D.	615.8	4.8

Henderson found the refraction (by 29 Cape observations of γ Draconis) = 614.10, which, when increased for the difference between Ivory's constant, and Bessel's reduced to the Cape, would become 617.86.

The arithmetical mean = 620·05, the geometrical = 620.03.

Ivory has given a table constructed on the hypothesis of U T for T = 70 and B = 28.85, from which I take, at zen. dist. 86°,

U. T.	653.1	6.5
Ivory	646.6	
U. D. D.	642.5	4.1

Arithmetical mean = 647.80, geometrical 647.77.

* As corrected by Plana (Observations, Int. lxxxvi.) The series for U T is slowly convergent, and the computation would be very troublesome, were it not for the tables of the integral which Bessel gives in the Fundamenta.

Again, zen. dist. 87°,

U. T.	802.5	12.4
Ivory	790.1	
U. D. D.	776.1	14.0

Arithmetical = 789.30 ; geometrical = 789.19.

Lastly, Brinkley gives the comparison of 42 observations of α Lyræ S P with these hypotheses, zenith distance = 87°.42′, T = 35°, B. 29.50,

U. T.	1067″.0	20.5
Observed	1046 .5	
U. D. D.	1011 .0	35.5

Arithmetical = 1039″ ; geometrical = 1038·6. But it must be remarked, that the temperature is by the internal thermometer, the external being 31.3; the barometer also is 0ⁱ.078 too little; in respect of the first of which the observed refraction should be lessened 9″.2, and for the second 2″.90.

It is evident that these means are not in error *one-twentieth of the difference between the two hypotheses ;* and, therefore, as far as 85° from the zenith may be depended on as certainly as any table extant.

Laplace used this principle not only in constructing the French tables, but also to show that the refractions above 74° are independent of the law of density. Brinkley, however, showed that the same method could assign them as far as 80°.45; the most important of the terms omitted by Laplace in the development of R in tang. θ has at that zen. distance in the two hypotheses the values 2″.60 and 1″.73; the arithmetical mean of these *cannot* be 0″.43 wrong, and its error is probably less than 0″.04. The opinion expressed by this great astronomer in his second memoir on refraction, Transactions Royal Irish Academy, vol. xiii. p. 169, that, by the method given there, a table of refractions could be more certainly derived from observation "than from any hypothesis respecting the actual variation of density," probably hindered him from pursuing the present method to its full extent, which, however, may be done with extreme facility.

In his notation, Transactions Royal Irish Academy, vol. xii. p. 83, the equation of refraction is,

$$d\text{R} = \frac{-d\rho \times ab \sin\theta \sqrt{1+b\rho'}}{2r(1+b\rho)\sqrt{1+b\rho-\frac{a^2}{r^2}(1+b\rho')\sin^2\theta}}$$

when ρ is the density at the distance r from the centre, ρ' and a, the same quantities at the earth's surface;* $b\rho$ the refractive force of air at the density ρ, and θ the apparent zenith distance.

If we assume,

$$\text{A} = \frac{\sqrt{1 + b\rho'} \sin \theta}{\sqrt{1 + b\rho - (1 + b\rho') \sin^2\theta}}$$

Q = refraction if the earth were plane,

$$s = \frac{r - a}{r},$$

Brinkley has shown, page 85, that,

$$d\text{Q} = \frac{-\frac{1}{2} b\text{A} d\rho}{1 + b\rho}$$

$$d\text{R} = \frac{(1 - s)\, d\text{Q}}{\sqrt{1 + (2s - s^2) \times \text{A}^2}}$$

and by developing A we find,

$$\text{A}^n = (\text{tang}\ \theta)^n \times \left\{1 + \frac{n}{2} \frac{b(\rho' - \rho)}{\cos^2\theta}\right\},$$

omitting higher powers of b. Developing dR we have,

* These quantities more strictly relate to the osculating circle, and the constant of a table must be modified accordingly. The quantity $\frac{l}{a}$ is one of these; if we assume the mean radius of curvature as the standard, and the earth's compression $\frac{1}{300}$, then for another latitude,

$$\frac{l}{a'} = \frac{l}{a} \times 1 + 0.0004991 \times \cos 2\text{L}.$$

Laplace has remarked that this should make the refraction to the north and south unequal. In fact, if we suppose the last rays of twilight to be once reflected, and that refraction ceases with reflection, (in which case I find, taking into account the curvature of the ray, which Delambre has neglected, that the height of the reflecting point is 41.536 miles,) and the ray is acted on in the case of horizontal refraction, through 8° 43′ of latitude. The change of the radius of curvature, and the place of its centre, must make a sensible difference in the two refractions, but the effect of the difference of temperature in the two trajectories is perhaps still greater.

The value of l is also inversely as local gravity, and that of b (or of the density corresponding to a given barometric column) directly as it; they must therefore be divided and multiplied respectively by $1 - 0.002695 \times \cos 2\text{L}$.

These corrections may seem minute, but are very sensible in low refractions.

$$d\text{R} = d\text{Q}$$

$$+\frac{\frac{1}{2}bd\rho}{1+b\rho}\times\begin{Bmatrix} s(\text{A}+\text{A}^3) - \frac{3}{2}s^2(\text{A}^3+\text{A}^5) \\ +\frac{1}{2}s^3(\text{A}^3+6\text{A}^5+5\text{A}^7) \\ -\frac{5}{8}s^4(3\text{A}^5+10\text{A}^7+7\text{A}^9) \\ +\frac{3}{8}s^5(\text{A}^5+15\text{A}^7+35\text{A}^9+21\text{A}^{11}) \\ \&\text{c.} \end{Bmatrix}$$

From the height of the atmosphere given in the preceding note $= 7.53 \times l$, it appears that b^2s is nearly $= s^5$, and, therefore, we need not develope beyond terms of this order, and the equation becomes

$$d\text{R} = d\text{Q}$$

$$+\tfrac{1}{2}bd\rho\times\begin{Bmatrix} s\times\frac{\text{tang}}{\cos^2}.\,\theta\,[1+\frac{1}{2}b(\rho'-\rho)\,(1+3\,\text{tang}^2.\,\theta)] \\ -\frac{3}{2}s^2\times\frac{\text{tang}^3}{\cos^2}.\,\theta\,[1+\frac{1}{2}b(\rho'-\rho)\,(3+5\,\text{tang}^2\,\theta)] \\ +\frac{1}{2}s^3\times\frac{\text{tang}^3}{\cos^2}.\,\theta.\,[1+5\text{tang}^2.\theta+\frac{1}{2}b(\rho'-\rho)\,(3+30\text{tang}^2+35\text{tang}^4)] \\ -\frac{5}{8}s^4\times\frac{\text{tang}^5}{\cos^2}.\,\theta[3+7\text{tang}^2\theta+\frac{1}{2}b(\rho'-\rho)(15+70\text{tang}^2\theta+63\text{tang}^4\theta)] \\ +\frac{3}{8}s^5\times\frac{\text{tang}^5}{\cos^2}.\,\theta\,[1+14\,\text{tang}^2+21\,\text{tang}^4\,\theta+\frac{1}{2}b\,(\rho'-\rho)\,(5+ \\ 105\,\text{tang}^2\,\theta+315\,\text{tang}^4+231\,\text{tang}^6)]. \end{Bmatrix}$$

These terms are of the form $s^n d\rho$, and $s^n\rho\, d\rho$.

The hypothesis of uniform temperature is expressed by the equation,

$$\rho = e^{-\frac{as}{l}},$$

giving the density unity at the surface, and evanescent at an infinite height. Between these limits we have,

$$\int_1^0 s^n d\rho = -\frac{l^n}{a^n}\times(n\,.\,n-1\,.\,.\,.\,.\,.\,2\,.\,1)$$

$$\int_1^0 s^n\rho d\rho = -\frac{l^n}{a^n}\frac{(n\,.\,n-1\,.\,.\,.\,.\,.\,.\,.\,.\,1)}{2^{n+1}}.$$

The hypothesis of uniformly decreasing density gives,

$$\rho = 1 - \frac{as}{2l}$$

$$\int_1^0 s^n d\rho = -\frac{l^n}{a^n} \times \frac{2^n}{n+1}$$

$$\int_1^0 s^n \rho d\rho = -\frac{l^n}{a^n} \times \frac{2^n}{(n+1)(n+2)}.$$

The term $\int s d\rho$, is the same on either hypothesis, being a result of the atmosphere's equilibrium; the coefficients of the higher terms differ, those on the hypothesis U T increasing much more rapidly. $\int s^2 d\rho$ is that which Brinkley added to Laplace's expression, using the arithmetical mean, which gives $\frac{5}{3} \times \frac{l^2}{a^2}$. I have preferred the geometric mean of the separate terms, as giving less weight to U T, which is especially necessary near the limit of convergence.* If we develope Q, pass from sines to arcs, and put μ for $\frac{\sqrt{1+b}-1}{\sin 1''}$, we shall have,

$$\text{R}'' = \mu \times \text{tang}\,\theta$$

$$+ \frac{\mu^2 \sin 1''}{2} \times \text{tang}^3\,\theta + \frac{\mu^3 \sin^2. 1''}{2} \times \text{tang}^5\,\theta \qquad (\text{Q}'.\ \text{Q}'')$$

$$- \frac{b}{\sin 2''} \times \frac{l}{a} . \frac{\text{tang}}{\cos^2}\,\theta\,[1.00000 + b \times \text{tang}^2\theta\,(1.06698)] \qquad (\alpha.\ \alpha')$$

$$+ \frac{b}{\sin 2''} \times \frac{l^2}{a^2} . \frac{\text{tang}^3}{\cos^2}\,\theta\,[2.44949 + b \times \text{tang}^2\,\theta\,(5.04119)] \qquad (\beta.\ \beta')$$

$$- \frac{b}{\sin 2''} \times \frac{l^3}{a^3} . \frac{\text{tang}^5}{\cos^2}\,\theta\,[8.65117 + b \times \text{tang}^2\,\theta\,(26.92202)] \qquad (\gamma.\ \gamma')$$

$$+ \frac{b}{\sin 2''} \times \frac{l^4}{a^4} \frac{\text{tang}^7}{\cos^2} . \theta\,[38.43867 + b \times \text{tang}^2\,\theta\,(160.08103)] \qquad (\delta.\ \delta')$$

$$- \frac{b}{\sin 2''} \times \frac{l^5}{a^5} . \frac{\text{tang}^9}{\cos^2} . \theta\,[199.22000\ \&c.].$$

* The original intention was to have assumed the terms $= \sqrt{a\text{I} \times a'\text{I}'}$; a and a' being arbitrary factors determined by observation; but as the simple $\sqrt{\text{I} \times \text{I}'}$ was found to satisfy my observations,

The terms β, γ, and δ have nearly the ratio $\frac{4l}{a} \times \text{tang}^2\,\theta$, and therefore the convergence ceases when the fraction $= 1$; or below 85°. Near that limit several of the higher terms are equal with opposite signs, and therefore (Lacroix, III. p. 160) I retain half the two last, which I find give at 85° the same results as a much more extended development, including all affected with b^3 and $\frac{\text{tang}^{13}}{\cos^2}\,\theta$.

This expression may be put into the form given by Brinkley, certainly the most convenient with which I am acquainted,

$$\text{R} = \mu \times \text{tang}\,\theta - \text{C};$$

the last of which quantities can be tabulated with the argument zenith distance, and is, in most cases, independent of the barometer and thermometer.

Their influence is, when necessary, easily allowed for: if a unit of air at 50° become $1 + \epsilon\,(t - 50)$ at $t°$, the quantity $\frac{l}{a}$ must be multiplied by this factor, and that of μ or b divided by it, from which we deduce the change of C for temperature,

$$\text{D} = \epsilon\,(t - 50°) \times [\alpha' + \beta - 2\text{Q}' - 3\text{Q}'' - \gamma],$$

which is always small from the absence of α, the largest of the terms.

this was unnecessary. Assuming Bessel's μ to be 57''.524, and Ivory's 58''.496, my table, when changed for these values, gives at their normal circumstances,

Zen. dist.	R — B.	R — I.
77°	—0''.11	—0''.02
78	—0 .10	—0 .05
79	—0 .11	—0 .07
80	—0 .12	—0 .10
81	—0 .06	—0 .12
82	—0 .08	—0 .19
83	—0 .10	—0 .25
84	—0 .13	—0 .30
85	—0 .28	—0 .42

The difference obviously depending on some slight difference between the values of μ and those used in computing the tables. It is equally evident, that to the zenith distance of 85 the results of the three formulæ are identical for all practical purposes.

If the barometer become $\text{H} + h$, instead of H, the normal pressure, the terms α, β, γ, &c., are to be multiplied by $\frac{\text{H}+h}{\text{H}}$; Q', α', β', &c., by its square, and Q'' by its cube; we find the barometric change of C,

$$\text{E} = \frac{h}{\text{H}} \times [\text{C} + \text{Q}' + 2\text{Q}'' - \alpha' + \beta' \text{ \&c.}].$$

If h be one inch, the value of E at $85^\circ = -2''.34$, so that these corrections can be worked by mental computation.*

* This form of the refraction has the advantage of being easily applicable to the equatorial. In a memoir on this instrument, (Trans. R. I. A. vol. xv.,) I have shewn that most of its corrections depend on an arc of the hour circle passing through the star intercepted between the pole and a perpendicular from the zenith. It is also equal to the intercept between the horizon and equator, whence I call it the horizontal declination. Denoting it by the symbol ζ, the polar distance by D; and being satisfied with the approximation, Refr. in P. Dist. = Refr. in Zen. Dist. $\times$ cosine of angle of position, we have,

$$(\text{R}) = \mu \times \text{tang}(\text{D} - \zeta) - \text{C} \times \frac{\text{tang}(\text{D} - \zeta)}{\text{tang}\,\theta}.$$

C may be put in the form,

$$\frac{\text{tang}}{\cos^2} \cdot \theta\, [q' \sin^2\theta - a + b\, \text{tang}^2\theta - c\, \text{tang}^4\theta \text{ \&c.}],$$

and its resultant in declination,

$$(\text{C}) = \frac{\text{tang}}{\cos^2}(\text{D} - \zeta) \times \frac{\cos^2\zeta}{\sin^2 \text{lat}} \times \left\{ \begin{array}{l} [q' \sin^2(\text{D} - \zeta) - a + b\,\text{tang}^2(\text{D} - \zeta) - c\,\text{tang}^4(\text{D} - \zeta)] \\ + q' \cos^2(\text{D} - \zeta)\left(1 - \frac{\sin^2 \text{lat}}{\cos^2\zeta}\right) \\ + \frac{\left(\frac{\cos^2\zeta}{\sin^2 l} - 1\right)}{\cos^2(\text{D} - \zeta)} \times \left[b - c\left(2\,\text{tang}^2(\text{D} - \zeta) + \frac{\left(\frac{\cos^2\zeta}{\cos^2(\text{D} - \zeta)} - 1\right)}{\cos^2(\text{D} - \zeta)}\right)\right] \end{array} \right\}$$

The first of these three terms is obviously the value of C taken with the argument $(\text{D} - \zeta)$ instead of θ, and multiplied by $\frac{\cos^2\zeta}{\sin^2 \text{lat}}$, of which latter a table for each hour is sufficient. The second is never $= 0''.01$; and the third, which is insensible above 80°, is computed by the formula

$$\frac{\text{tang}}{\cos^4}(\text{D} - \zeta)\frac{\cos^2\zeta}{\sin^2 \text{lat}} \times \left(\frac{\cos^2\zeta}{\sin^2 \text{lat}} - 1\right)\left[\log^{-1}(6.28162) - \log^{-1}\frac{(3.90574)\left(\frac{\cos^2\zeta}{\sin^2 l} + 1\right)}{\cos^2(\text{D} - \zeta)}\right],$$

which at 85° zenith distance and 6 hours from the meridian, is only $1''58$, and (if it be thought

To construct a table of refractions from this formula, we require the numerical values of $\frac{l}{a}$, of μ at some given temperature and pressure, and of ϵ the expansion of air for one degree of Fahrenheit. The last of these has almost universally been taken from Gay Lussac, who found that a unit of any gas or vapour at the freezing point of water, became 1.375 at the boiling point. But the experiments of Rudberg have shown that this number is too great, and that the true increase is 1.365. I have, therefore, used this coefficient, notwithstanding the opinion of some whose authority is of much weight, that even Gay Lussac's number should be increased on account of the moisture of the atmosphere. But the expansion of vapour is the same as of dry air: if water be present, it does indeed seem greater, because heat increases the quantity as well as the bulk of the vapour, and a correction to this effect is necessary to the barometric measurement of heights. In respect of refraction the case is otherwise; aqueous vapour and dry air refract alike under equal pressure and temperature; when, therefore, more vapour is added to the atmosphere, the effect is the same as if so much dry air were added as is equivalent to its tension. Observation leads to the same conclusion; for the illustrious astronomer of Königsberg found that the coefficient which satisfies the variations of refraction is 1.00364.—Tab. Reg. p. lx. The only way in which the hygrometric state of the atmosphere can affect refraction is by changing the value of l, or by varying the arrangement of the strata. The latter of these cannot be taken into account, and the former is, in this climate, insensible within the limits of this inquiry.

The value of l used is that given by Arago and Biot in their experiments on the refractive power of air. They give it for 0 centesimal; but as their experiments were made at the mean temperature 10° cent. or 50° Fahrenheit, the normal temperature of most refraction tables, their result is not affected by the error of Gay Lussac's expansion.

There remains only the refractive power of air, which may be investigated

necessary to employ it) can be computed by the sliding rule. A table of ζ for every minute of the first 6 hours is almost essential to the use of the equatorial, and if my first table and the second $\times \frac{\cos^2\zeta}{\sin^2 l}$ were added to it, the refraction can be as easily computed as on the meridian.

either by direct experiment, as was done by Arago and Biot,* or by astronomical observations. Notwithstanding the well known accuracy of these distinguished philosophers, it seems desirable that their conclusions should be verified by the more refined means of examination, which Arago himself has since indicated. At present, the result appears in excess, giving for μ at 50° and 29^{i}.60 the value 57″.82. That which is most generally received is De Lambre's, employed in the French tables, as well as in those of Brinkley and Ivory. It is at the same temperature and pressure 57″.72, and was deduced from observations made with the repeating circles of Le Noir, so that it would not have much weight now were it not for the confirmation which it seemed to derive from the comparison of simultaneous observations by Brinkley and Brisbane, at Dublin and Paramatta. The sum of the Dublin north polar, and Paramatta south polar distances gives very nearly 180 degrees, and the resulting value of μ is 57.77; but it must be remarked, that the temperature used in computation is that by the internal thermometer, which, however necessary at Dublin, may not be so at the other observatory. It is also important to notice, that the Dublin barometer is by no means perfect. I have been enabled to determine its error by comparison with that of the Magnetic Observatory of Trinity College, (by Newman, and differing from mine and the standard of the Royal Society merely in having the cistern of glass.) Observations made during thirteen successive days at 22^{h} give

	BAR.	E. T.	A. T.
Magnetic Observ. .	30.001	41.60	41.60
Astronom. Observ. .	29.625	35.53	37.70

The difference of height of these stations is, according to Captain Larcom, 258.8 feet, and I compute that the actual pressure at the upper station was 29.702; so that the reading there requires the correction + 0.077. Subsequently this has been confirmed by the kindness of Dr. Coulter, who compared two portable barometers, by Cary, with that of the magnetic observatory, very carefully. They were then carried out to the astronomical observatory, compared there, and on their return compared again with the magnetic. From the result of the two sets I deduce the corrections + 0.0770, and + 0.0800, the mean + 0.0785 I consider preferable to the other, and this would reduce the constant 57.72 to

* Memoires des Scavans Etrangers, T. vii.

57.567, a remarkable approximation to that of Bessel. This is, however, for the temperature of the barometer 37° ; but it will probably avail for 50° also ; as if, on the other hand, the Dublin barometer has a wooden mounting, on the other there is probably a little air in the upper part of the tube which will compensate for its inferior expansion of scale.

Bessel has given for a or $\frac{\frac{1}{2}b}{1+b}$, 57.538 at 48°.75, but the barometer at 50°. He, however, found afterwards, that the refractions of his table require to be multiplied by 1.001779, which would make it at the normal temperature and pressure 57.4993, hence $\mu = 57.524$. This appears to satisfy the Greenwich observations, as well as* those at the Cape of Good Hope; and its unexpected agreement with Brinkley shows how safely it may be depended on. At the same time, the very circumstances of that agreement give additional weight to the opinion which I have already expressed, that every fixed observatory should verify the refractions which it employs, and employ meteorological instruments of the best quality that can be made.

The *observed* refraction of a star below the pole is obviously (omitting degrees)

$$\text{R} = \text{O} - \delta,$$

O being the observed polar distance, δ the assumed declination of the star. Calling dO and $d\delta$ the corrections which these require, the true refraction is

$$\text{O} - \delta + d\text{O} - d\delta.$$

If we put $\mu \times \text{V}$ for the tabular refraction, we have,

$$\text{V}\,(\mu + d\mu) = \text{R} + d\text{O} - d\delta,$$

Now, the polar point having been determined with an erroneous refraction, all the polar distances require the correction $d\mu \times \text{P}$; and if we determine the declination by observations above the pole, we have,

$$d\text{O} = d\mu \times \text{P}; \;\; d\delta = -\, d\mu\,(\text{V}' + \text{P});$$

and hence,

$$\text{R} - \text{V}\mu = d\text{R} = d\mu\,[\text{V} - \text{V}' - 2\text{P}] = d\mu \times \text{K}.$$

* When the necessary corrections for the latitude and the change of the length of the pendulum are applied.

The constants V and V′ must be computed for the mean refraction of each set of observations; P from the annual mean temperature and pressure, as the observations for index correction and latitude extend through the year.

If we observe a star of southern declination, and assume it to have been well determined at some place where it passes near the zenith, we obtain $d\mu$ with a much larger coefficient, for we find in the same way,

$$dR = d\mu\,(V + P) = d\mu \times K.$$

It may be doubted, however, whether anything is gained by the superior magnitude of K; for it is unsafe to argue, as if the results of one set of instruments were identical with those which another would give in the same locality. The refraction used at the southern observatory must also have been carefully verified, as P′ the polar constant is in those existing very considerable.

The process must, of course, be applied to as many stars as possible, both for the sake of accuracy in the final result, and also because the identity of the values of $d\mu$, obtained at different zenith distances, is an evidence of the correctness of the formula used to compute the refraction. Among the various modes of combining the partial results, I prefer that which makes the sum of the squares of errors of observation a minimum; not taking into account those irregular fluctuations to which low stars are liable, caused by momentary changes in $d\mu$, or in the law of density, and, therefore, scarcely coming within this application of the theory of probabilities.* This gives the formula,

$$d\mu = \frac{K \times S\,(dR) + K' \times S\,(dR') \ldots.}{K^2 \times n + K'^2 \times n' \ldots.}$$

The Armagh circle has been described by me in the Memoirs of the Royal Ast. Soc. vol. ix. After using it pretty extensively, during the last six years, I have found no reason to change the favourable opinion of it which is expressed there; and, in particular, find no trace of the evil which Mr. Airy considers probable in circles divided on the face, namely, great and irregular fluctuations of run in the microscopes, (Mem. R. Ast. Soc. vol. x. p. 266.) So far from this, it is remarkably steady in that respect. A change of 30° alters the mean run of the four microscopes from 0″.25 to 0″.47; the utmost force that can be applied

* See on this subject, Bessel Ast. Nachrichten, No. 358.

drawing the instrument from the pier, and pushing it toward it, makes only a change of 0″.02; of 30 sets taken round the circle at different times, the greatest I have found is 0″.75, and the least 0″.00; and during the last three years that at 360° (which equals the mean of the 30 sets) has been within the limits of 0″.25 and 0″.54. In respect of its division, after a careful examination of 288 diameters in four positions, I have stated, that I considered it good; trifling, however, as the resulting error may be, it is obviously always necessary to correct for it when it is known. I have not, however, obtained my corrections in the present instance by the method described in that memoir. The errors which I found were absolutely casual, so that it was impossible to interpolate between them; the individual research of each would have demanded an impracticable sacrifice of time; and even could this have been afforded, the value of the result appears to me at least doubtful. All such modes of examination assume, that the divisions keep the same relative position while the circle is turned through any arc; but it is found in actual experience, both with this and other circles, that occasionally the correction of a diameter varies with its situation to a whole second or even more. I have, therefore, applied twelve equidistant microscopes to the circle; and presuming (as is also shown by the table of errors which I had constructed by my first method of correction) that their mean is free from sensible error, I use it to correct that of the four reading microscopes, in a way as simple as I believe it to be effective. Let M_x, m_x be the means of the reading microscopes, and of the twelve when any number x is at the index. Then, on this supposition, we have,

$$m_x - m_o = \mathrm{M}_x - \mathrm{M}_o + \epsilon(x) - \epsilon(o).$$

We may assume the reading of the four at o to be a zero to which all others are referred, and therefore,

$$\epsilon(x) = (m_x - m_o) - (\mathrm{M}_x - \mathrm{M}_o),$$

which only implies the permanence of the microscopes while the readings are taken. Out of more than 100 of these corrections most are negative, which arises from the zero reading M_o requiring, according to my former mode of examination, a correction of $+0''.93$; about one-fourth of the number differ from this more than ± 0.49, and in some I have found reason to suspect a minute change depending on the temperature. As, however, it can be deter-

mined in a few minutes at the very time of observation, this is of no consequence.

The index correction of this instrument is deduced from observations of Polaris. The star is observed five times near the meridian, and reduced to it by a table computed from the formula,

$$do = \text{A} + \text{A}^2 \times \text{tang}\,\delta \times \sin 1'',$$

where,

$$\text{A} = \frac{\sin \times \cos . \delta}{\sin 1''} \times \text{versine P}.$$

These, compared with the mean places of Bessel brought up by the constants of Baily's catalogue (for the time) and corrected for the term 2☽, give the approximate correction. When conjugate observations (above and below the pole) can be obtained, the mean is independent of any error of the assumed declinations; but at other times the difference between Bessel's place and my own is applied as a correction.* As long as the difference of individual results is manifestly mere error of observation, it is assumed that the mean is the index correction during that period. Its changes are slow, having an annual period, and a given extent of variation during the eight years that the instrument has been used. The most probable cause of this appears to be some influence of temperature on the hill, for the transit instrument, and a telescopic meridian mark about fifty feet south, suffer analogous variations. As the fact is curious, I annex a table of the index corrections during 1839, which will also show that no error can arise from its occurrence.†

* Equal to $+ 0''.21$ by 700 conjugate observations.

† 1838, Dec. 18,	— 4.16	80 obs.
1839, Feb. 24,	— 4.75	40
„ April 7,	— 5.20	50
„ „ 24,	— 4.19	55
„ May 16,	— 3.27	115
„ June 3,	— 1.63	10
„ „ 25,	— 0.14	75
„ Sept. 11,	— 2.39	45
„ Oct. 18,	— 3.49	105
„ Dec. 29,	— 4.27	25
1840, Feb. 28,		

The declinations of those refraction stars which are in the Nautical Almanac were compared with its places, as long as they were given to the second place of decimals. Afterwards, they were reduced by the constants of Baily's catalogue, and compared with its mean places for the year, corrected when necessary for proper motion. The others were taken from that catalogue, and reduced by its precession, corrected for Bessel's last value of n, and for secular variation (computed from its value compared with the precessions given in the Fundamenta). When any of them have been observed at Greenwich, by Airy, the proper motion has been deduced from his results by the formula,

$$\pi = \frac{\text{A} - \text{cat} + \frac{4}{3}\,(\text{P} - \text{B}) - 1''.053 \times \cos a}{75 + t},$$

where P — B is the number found in the last column of the Fundamenta, t the time in years from 1830, and 1.053 the correction for the error in the constant of precession used in that work. When Airy had not observed the star, I use my own declination changed for Bessel's refraction.

The low stars are often neat spectra (that of α Lyræ, I have found 22″ long); sometimes the blue and violet disappear for several seconds, and sometimes, though less frequently, the red, the rest remaining unabsorbed. When the colours are distinctly separated, I take the yellow where it borders on green, which I think a tolerable average for the mean of the spectrum. The star should be carefully watched during its whole transit, for the undulations that produce irregular refraction are often of long duration; and sometimes a star, which is apparently well bisected for several seconds, will leave the wire altogether.

The temperature is observed by a thermometer of Troughton which I found here. I have verified its freezing and boiling points to assure myself that it had not undergone the change said to have occurred in some thermometers. I have also compared it at several points with a standard instrument made for me by Troughton and Simms, in 1834, and think it of equal excellence. It is established at a north window of the eastern tower, about four feet above the centre of the circle, and twelve distant in a horizontal direction, in a semicylinder of polished copper, and an interior one of tin, arranged so as to permit a free circulation of air, but excluding all external radiation. In summer, when the rays of the sun reach the northern side of the tower, a second thermometer is used at a southern

window of the same tower, till both agree, which generally is the case an hour after sunset. The internal temperature is also in most cases recorded, from a third standard thermometer attached to the telescope near its centre; but in this observatory it is not to be used in computing refraction. If any error were produced by preferring the external, its amount should be greatest when the difference is greatest, which I do not find to be the case. For instance, among 39 refractions of α Cygni, I find,

9 with I — E from 0° to 3°, mean 2°.37, give diff. from mean — 0″.22.
10 from 3° to 4° diff., mean 3°.39, give — 0″.17
10 from 4° to 5° diff., mean 4°.45, give + 0″.58
10 from 5° to 7°, mean 6°.01, give — 0″.21

In this star, 1° would change the refraction 0″.72.

Among southern stars, 23 of λ Sagittarii.

8 from 0° to 3° mean 2°.16 give — 0″.22
8 from 3° to 5° mean 3°.78 give — 0″.11
7 from 5° to 7° mean 5°.66 give + 0″.33

Here 1° gives a change of 0″.65. In these the discordances obviously have no connexion with the state of the internal thermometer; and the case is the same with other stars.

The barometer used was, till December 4, 1835, a portable one, by Ramsden. It was then replaced by a standard one of Newman, similar to that described by Mr. Baily in the Philosophical Transactions for 1837, p. 431. Mr. Newman states, that the specific gravity of its mercury is 13.545 at 60°, and that the diameter of its tube is $0^{i}.570$. In such a tube the correction for capillary action is nearly insensible; but it happens to be unnecessary here, for a reason given by Laplace, Conn. des Tems, 1829, but not, that I am aware, noticed in any English work. In barometers like this, the scale is terminated at its lower extremity with a point which is brought into contact with the mercury of the cistern; but the surface of the latter is also curved, so that the contact, if near the edge, is made at a surface lower than the real zero. If the distance from the edge be properly assumed, this may be made to counteract the depression above: it is rather too great here, giving only $0^{i}.003$, but the rest is neutralized by the fact, that the contact (if estimated, as I do it, by the meeting of the point

and its reflected image) does not take place without a minute depression of the mercury, which is between 0.001 and 0.002.

The refractions have been computed with $\mu = 57.7682$ (Brinkley's reduced to my latitude), and the colatitude 35° 38′ 47″.3. In this climate and this exposed situation, it is not very easy to observe by reflection, and I have not yet definitively settled this element.

With the first division of the circle,		41	pair give	47″. 22
With the second	,,	58	,,	47″. 48
With the third	,,	132	,,	47″. 37
		mean	.	47″. 37

The first and third are corrected for error of division. In the second, three divisions were read at each microscope. It is obvious that these give no reason for changing 47″.3, which had previously been determined with Troughton's equatorial by upwards of 200 pair of observations; and equally so that whatever uncertainty there be, can have no effect.

The following are the results that I have obtained:

45 ω^2 *Cygni.*

Twelve observations (1838. 772) with Brinkley's Constant of Refraction give the Declination for 1830,

$$\delta = +48° \; 23' \; 1''. 51.$$

Precession $= + 11''. 844$; sec var. $= + 0''. 212$; proper motion $= + 0''.033$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.*			OBS. REFRACT.	dR.
1836,	Feb.	14.	42.2	43.5	44.2	30.122	77°	10′.	53	256.67	+ 0.01
,,	,,	17.	36.2	38.3	39.1	30.241	77	10	55	256.51	— 4.47
,,	,,	26.	29.7	34.5	35	28.979	77	10	65	252.63	— 0.77
1838,	Feb.	7.	37.0	39.5	40.1	29.804	77	10	27	253.50	— 3.07
,,	,,	8.	37.5	39.9	41.4	30.173	77	10	27	255.00	— 4.58
,,	,,	15.	38.8	44.1	45	29.768	77	10	36	250.60	— 4.70
,,	,,	17.	35.5	39.3	40.6	29.367	77	10	35	251.06	— 2.62
,,	,,	23.	31.2	35.6	37.1	29.474	77	10	28	256.20	— 0.76
,,	,,	29.	43.8	46.8	48.3	30.409	77	10	29	256.63	— 1.44

* The figures after the minutes of zenith distance are decimals.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1839, Feb. 9.	37.1	..	43.1	30.084	77° 9'. 89	257.20	— 1.64
" " 12.	36.7	..	40.9	30.046	77 9 90	257.59	— 1.19
" " 14.	35.8	..	40.5	29.733	77 9 93	256.58	— 0.33
" " 17.	34.2	..	39.5	29.915	77 9 95	261.75	+ 2.73
" " 18.	31	..	34.2	29.380	77 9 96	255.97	— 0.17
" " 24.	33.1	..	37.4	29.462	77 10 02	253.11	— 2.59
" April 5.	40.9	..	43.7	29.735	77 10 12	253.17	— 0.72
" " 7.	38.1	40.9	42.2	30.091	77 10 07	256.50	— 1.94

$17 \times d\text{R} = -28''.25$ $\qquad$ $d\text{R} = -1''.66$

$\kappa = 2.8861$ $\qquad$ $d\mu = -0.576$

31. *o Cygni.*

Twelve observations (1838. 533) give

$$\delta = +46^\circ\,13'\,45''.59.$$

Precession $= +10''.648$; sec var. $= +0''.228$; proper motion $= +0''.039$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, March 1.	29.2	34	35	30.193	79° 18'. 92	314.06	— 2.27
" " 14.	34.1	37.3	39.5	30.287	79 18 91	314.91	+ 0.98
" " 23.	32.2	34.5	35.8	29.665	79 19 05	307.47	— 1.38
" " 24.	29	33.3	36.8	29.725	79 19 05	307.50	— 4.04
" " 30.	36.1	38	42.1	29.758	79 19 03	309.29	+ 2.16
" April 3.	35	37.8	39	29.429	79 19 03	308.90	+ 4.41
" " 4.	35	38	40.3	29.683	79 19 11	304.79	— 2.36
" " 7.	38.9	41.7	43.2	30.297	79 19 03	309.08	— 1.77
1838, Feb. 20.	31	34.4	35	29.496	79 18 51	305.76	— 1.88
" " 21.	31.8	35.5	36.6	29.577	79 18 78	307.62	— 0.41
" March 6.	38.8	39.7	40.2	29.456	79 18 93	301.81	— 0.42
" " 7.	36.5	38.8	40.3	29.790	79 18 86	305.76	— 1.42
" " 8.	37.9	39.9	41.7	30.176	79 18 79	310.09	— 0.10
" " 17.	35.8	39.1	40.9	29.368	79 18 95	301.48	— 1.78
" " 23.	31.3	35.7	37.3	29.480	79 18 88	309.24	+ 1.86
" " 29.	44.2	47	48.5	30.410	79 18 83	309.43	+ 1.04

$16 \times d\text{R} = -7''.38$ $\qquad$ $d\text{R} = -0''.46$

$\kappa = 3.7450$ $\qquad$ $d\mu = -0.160$

Capella.

Eighteen observations (1837. 65) give,

$*\delta = + 45° 48' 54''. 12.$

Precession $= + 4.840$; sec var. $= - 0''.627$; proper motion $= - 0''.472$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, June 22.	58.3	63.6	64.8	30.114	79° 44'. 17	307.47	— 0.54
„ „ 25.	50	56.9	59	30.076	79 44 10	311.98	— 0.43
„ July 7.	55.8	60.3	62	30.100	79 44 15	309.64	+ 0.23
„ „ 8.	58.8	62.9	65	30.019	79 44 18	308.36	+ 1.57
„ „ 9.	55.3	62	64	29.899	79 44 16	309.66	+ 1.91
„ „ 13.	56.4	61	64.5	29.472	79 44 28	302.94	+ 0.19
„ „ 14.	58.3	65.4	64.2	29.544	79 44 11	302.49	+ 0.34
„ „ 16.	58.8	60.6	62.3	29.917	79 44 22	306.27	— 1.46
„ „ 26.	60.1	62.1	64.1	29.762	79 44 32	301.23	— 2.22
„ „ 27.	55.4	59	62	29.571	79 44 30	301.86	— 2.56
„ August 5.	48.9	53	55.9	30.150	79 44 11	313.34	— 1.24
„ „ 6.	52.8	56.9	59	30.239	79 44 17	311.43	— 1.55
„ „ 7.	53.5	57.8	60.3	30.264	79 44 15	311.58	— 1.28
„ „ 8.	56.1	59.5	61.8	30.193	79 44 18	309.35	— 1.02
„ „ 14.	58.2	61.3	64	30.069	79 44 21	307.58	— 0.09
„ „ 15.	60.9	62.5	64.5	30.079	79 44 25	306.22	+ 0.03
1838, July 25.	52.1	..	59	29.897	79 44 11	307.74	— 2.08
„ „ 26.	52.4	57.1	58.5	29.678	79 44 20	303.13	— 4.27
„ August 4.	56.7	..	62	29.203	79 44 24	299.29	— 0.56
„ „ 5.	54	58	60	29.008	79 44 27	298.63	— 0.83

$20 \times d\text{R} = - 15''.86$ $\qquad d\text{R} = - 0''. 79$

$\text{K} = 3.7318$ $\qquad d\mu = - 0.\ 212$

* Brinkley's δ	= 54''.70	Airy (Cambridge,) . .	54''.78
Bessel's, ...	53 .61	Argelander, . . .	53 .50
Airy (Greenwich,) . .	53 .40	Mine, ...	54 .31

P.XXI. 157 *Cygni.*

Fifteen observations (1838. 800) give,

*δ for 1838. Jan. 1, $= + 45° 42' 55''.74$.

Precession $= + 15''.586$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, March 13.	29.2	33.6	34.8	30.211	79° 50'. 72	329.51	— 2.93
„ „ 14.	33	36.8	38	30.277	79 50 69	331.53	+ 1.14
„ „ 24.	28.2	33	35.1	29.726	79 50 81	325.78	— 1.96
„ „ 29.	32.1	35	38	29 535	79 50 84	324.26	+ 1.31
„ „ 30.	34.7	36.5	42.1	29.760	79 50 82	326.20	+ 2.68
„ April 1.	33.8	35	39	29 810	79 50 85	324.39	+ 0.29
„ „ 3.	33	37	38	29.438	79 50 92	320.41	— 0.91
1838, „ 11.	40.5	45	46	29.849	79 50 64	321.39	+ 0.99

$8 \times d\text{R} = + 0''.62$ $\qquad$ $d\text{R} = + 0''.077$

$\kappa = 4\ .0544$ $\qquad$ $d\mu = + 0\ .019$

22. *Andromedæ.*

Eleven observations (1838. 337) give,

$\delta = + 45° 7' 33''.65$.

Precession $= + 20''.056$; sec var. $= - 0''.009$; proper motion $= + 0.''005$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, May, 3.	44.7	46.2	50	29.722	80° 23'. 54	331.12	— 2.21
1838, „ 4.	44.2	48.8	50.1	30.008	80 23 12	334.54	— 3.13
„ „ 5.	48.7	52	53.5	30.200	80 23 12	334.72	— 0.89
„ „ 6.	52.1	54	55.8	30.163	80 23 15	332.73	— 0.11
„ „ 8.	56.5	60	61.8	30.176	80 23 19	329.47	+ 0.56
„ „ 10.	47.1	53.2	55	30.260	80 23 06	338.20	+ 0.90
„ „ 11.	49.1	53.5	55.2	30.132	80 23 15	332.81	— 1.72
1839, April, 17.	37.8	40.8	43.4	29.101	80 22 82	328.10	— 2.83

* This star has not been reduced to 1830, as I am doubtful of Piazzi's place; the right ascension which he gives is also erroneous.

It is rather too faint for subpolar observation here.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1839, April, 18.	36.9	43.3	44.2	29.212	80° 22′. 77	331.71	— 1.05
„ „ 19.	40	42.7	43.9	29.766	80 22 70	336.39	— 0.38
„ „ 23.	46.8	50.5	51.3	29.916	80 22 79	331.63	— 2.02
„ „ 24.	44.4	47.4	49.2	29.912	80 22 73	335.20	— 0.05
„ „ 30.	50.6	53	54	29.818	80 22 87	327.36	— 2.54
„ May 2.	46.1	48.1	53	29.890	80 22 77	332.96	— 0.78
„ „ 7.	49.8	51	53.1	29.875	80 22 86	327.98	— 3.12
„ „ 10.	43.2	47.4	49.2	30.124	80 22 76	334.07	— 1.90
„ „ 12.	44.9	47.9	50.8	30.002	80 22 81	331.43	— 3.30

$17 \times d\text{R} = -24''.57$ $\qquad d\text{R} = -1''.44$

$\kappa = 4.1560$ $\qquad d\mu = -0''.348$

β Aurigæ.

Nine observations (1837. 675) give

$^{*}\ \delta = +44^\circ\ 55'\ 12''.66.$

Precession $= +1''.132$; sec var. $= -0''.642$; proper motion $= -0.''019$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1833, July 23.	49.9	56.6	..	29.718	80° 37′. 97	339.66	+ 2.07
„ August 1.	56.3	62.2	..	30.348	80 38 00	339.51	— 0.64
„ „ 2.	55.9	61	..	30.268	80 38 05	336.49	— 2 98
1835, July 29.	55.1	..	..	30.076	80 37 99	331.64	— 6.35
„ „ 31.	57.7	62	..	29.993	80 37 93	335.03	+ 0.71
„ Aug. 2.	58.2	62	..	29.871	80 37 98	331.45	— 1.66
„ „ 6.	53.8	61.6	..	29.796	80 37 91	336.55	+ 0.96
„ „ 30.	57.2	60.6	..	29.858	80 37 97	333.55	— 0.47
1837, July 8.	57.7	63	64.2	30.025	80 37 79	334.92	+ 0.52
„ „ 9.	54.1	61	63.0	29.896	80 37 73	338.34	+ 1.98
„ „ 10.	56.3	62.3	65	29.846	80 37 80	334.37	+ 0.22
„ „ 13.	56	59.8	63.1	29.454	80 37 89	329.11	— 1.10
„ „ 14.	57.7	65	64	29.544	80 37 85	331.49	+ 0.55

* Airy (Greenwich, 36 and 37) . . 11″. 40 Argelander . . 11″. 00
„ (Cambridge) . . . . 12 . 35 Mine . . . . 12 . 76

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, July 16.	55	60	61.8	29.908	80° 37′. 75	337.77	+ 1.72
,, ,, 20.	55 5	61	63	29.934	80 37 86	331.43	— 4.50
,, ,, 27.	55.8	58.2	61.5	29.571	80 37 88	330.75	— 0.80
,, August 5.	47.8	52	54.9	30.152	80 37 67	339.71	— 4.35
,, ,, 6.	51.5	54.9	57.2	30.239	80 37 75	339.43	— 2.82
,, ,, 7.	51.2	55.7	59	30.264	80 37 74	339.63	— 3.14
,, ,, 8.	55	58.9	61	30.193	80 37 79	336.78	— 2.43
,, ,, 14.	57.1	61	63	30.069	80 37 82	335.27	— 1.07
,, ,, 15.	58.8	61.9	63.1	30.081	80 37 81	333.83	— 1.49
,, ,, 16.	60.9	63	65	29.971	80 37 89	331.26	— 1.41
,, ,, 26.	50.9	55.8	59	29.930	80 37 71	339.54	+ 0.45
,, ,, 29.	48.2	54.9	57.3	29.429	80 37 86	333.88	— 1.49
,, ,, 31.	50.1	55	57	29.266	80 37 97	330.74	— 1.54
1838, July 25.	50.8	..	58	29.883	80 37 71	338.42	— 0.21
,, ,, 26.	51.2	..	57	29.680	80 37 75	336.39	+ 0.30
,, August 4.	55.7	..	61.5	29.205	80 37 86	330.05	+ 2.41
,, ,, 5.	53.1	..	59.1	29.013	80 37 98	323.32	— 4.03

$30 \times d\text{R} = -30''.59$ $\qquad$ $d\text{R} = -1''.02$

$\text{K} = 4.2046$ $\qquad$ $d\mu = -0.242$

a Cygni.

Twenty-four observations (1838. 105) give,

$*\delta = +44° 40' 35''.50.$

Precession $= +12''.597$; sec var. $= +0''.226$; proper motion insensible.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1836, Feb. 17.	36.2	38.2	38	30.241	80° 51′. 08	359.84	— 2.16
,, ,, 26.	29.7	34.5	35.5	28.985	80 51 40	348.59	— 3.39
,, March 7.	34	39.8	40.2	29.166	80 51 49	349.11	— 1.78
1837, March 12.	28.4	33.1	35	29.617	80 51 07	359.68	— 0.80
,, ,, 13.	29.2	34	35	30.193	80 51 02	362.03	— 4.85
,, ,, 17.	38.1	40.4	41.3	30.206	80 51 09	358.74	— 1.30
,, ,, 24.	28.6	34.9	36.8	29.722	80 51 12	357.98	— 3.63
,, ,, 29.	32	37.4	38.2	29.530	80 51 20	353.88	— 2.77

* Brinkley's δ . . . . = 36.25
Bessel, ... 34.21
Argelander, . . 35.50
Airy, Cambridge, . 35.14
Airy, Greenwich, (36), . = 34.76
Challis (1837,) . . . 35.95
Mine, ... 35.70

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, April 1.	34.6	38	40	29.816	80° 51′ 20	353.63	— 4.48
„ „ 3.	34	37.5	38	29.438	80 51 27	350.08	— 4.00
„ „ 4.	34.4	37.8	40.3	29.683	80 51 18	354.39	— 3.08
„ „ 7.	37.6	40.6	42	30.308	80 51 15	357.57	— 4.79
„ „ 9.	39	42.1	43.2	30.245	80 51 10	359.99	— 0.77
„ „ 16.	35	40.2	41	29.558	80 51 23	352.35	— 2.29
„ „ 17.	42	43.2	44.5	29.764	80 51 23	352.02	+ 0.18
1838, March 7.	37	39.5	40.1	29.804	80 50 96	352.08	— 3.93
„ „ 8.	37.3	39.6	41.4	30.173	80 50 86	358.00	— 2.10
„ „ 17.	35.2	39.6	40.3	29.366	80 51 00	351.13	— 0.77
„ „ 23.	31	35.5	36.9	29.468	80 50 93	356.10	— 0.45
„ „ 29.	43.8	46.8	48.3	30.409	80 50 93	357.18	— 0.67
„ April 11.	41.3	45.4	47	29.830	80 50 97	356.23	+ 3.14
„ „ 12.	43	46.1	47.6	30.188	80 50 94	357.43	+ 1.56
1839, Feb. 9.	37.1	..	43.1	30.084	80 50 48	360.18	+ 0.28
„ „ 12.	36.7	..	40.9	30.046	80 50 50	359.80	+ 0.97
„ „ 17.	24.2	..	28.2	29.244	80 50 58	356.36	— 0.74
„ „ 18.	31.7	..	34.2	29.374	80 50 62	354.75	— 0.03
„ „ 20.	28.9	..	33.5	30.066	80 50 44	365.35	+ 0.10
„ „ 24.	33.1	..	37.3	29.462	80 50 65	353.85	— 0.86
„ March 2.	37	..	44	29.856	80 50 66	354.79	— 1.53
„ „ 3.	40.2	..	43.6	29.820	80 50 71	352.37	— 1.18
„ „ 17.	34.2	..	39.5	29.915	80 50 67	357.23	— 2.06
„ „ 25.	36.6	..	42.9	29.424	80 50 83	348.45	— 3.13
„ „ 27.	41	..	45.4	29.082	80 50 93	341.86	— 2.40
„ April 5.	40.9	45	43.7	29.735	80 50 85	347.89	— 4.28
„ „ 6.	38.1	42.8	44.8	30.122	80 50 66	359.57	+ 0.93
„ „ 7.	38.1	40.9	42.2	30.091	80 50 69	357.61	— 0.78
„ „ 11.	39.9	43	46	30.442	80 50 62	361.85	+ 0.79
„ „ 12.	44.2	46.5	47.1	30,270	80 50 72	356.80	— 0.02
„ „ 19.	44.8	47	47.6	29.708	80 50 86	347.01	— 1.92

$39 \times d\text{R} = -58''.99$ $\qquad d\text{R} = -1''.51$

$\kappa = 4.5685$ $\qquad d\mu = -0.331$

46 *Andromedæ.*

Thirteen observations (1838. 083) give,

$\delta = +44° 38' 7''.08.$

Precession $= +19''.065$; sec var. $= -0''.161$; proper motion $= -0''.017$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, May 18.	45.1	49.9	50	30.193	80° 52′. 63	355.18	— 0.18
1838, May 5.	47.2	50.7	52.2	30.200	80 52 30	352.96	— 0.65
„ „ 6.	49.9	52.8	54.1	30.165	80 52 34	350.11	— 1.09

DATE.		E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1838,	May 8.	53.7	57.9	60.1	30.172	80° 52′. 36	349.05	+ 0.59
„	„ 10.	46.0	52.1	53.4	30.261	80 52 22	357.59	+ 2.47
„	„ 11.	47.1	52	54.5	30.128	80 52 32	351.68	— 1.09
„	„ 15.	39.4	45.2	47.7	29.688	80 52 31	352.44	— 0.97
„	„ 23.	48.2	56.7	54.5	29.780	80 52 45	344.70	— 3.26
„	„ 24.	49.2	53	54.7	29.864	80 52 36	350.03	+ 1.84
1839,	April 23.	45.8	48	50.2	29.912	80 52 02	349.51	— 1.63
„	May 2.	44.3	49	50	29.884	80 51 98	351.59	— 0.27
„	„ 6.	45	51	52.5	29.989	80 51 97	352.78	+ 0.30
„	„ 7.	46	49.9	51.3	29.864	80 52 05	347.81	— 2.63
„	„ 10.	41	45.8	48	30.136	80 51 88	358.50	+ 1.21
„	„ 12.	43	46.1	49	29.984	80 51 89	357.08	+ 3.10
„	„ 21.	44.8	50.2	52	30.050	80 52 00	351.48	— 1.94
„	„ 22.	42.7	46.2	49.2	30.176	80 51 92	356.59	+ 0.31
„	„ 25.	48	53.8	55.2	30.028	80 52 04	349.25	— 1.51
„	„ 26.	48.2	52	54.7	29.987	80 52 05	348.53	— 1.63

$19 \times d\text{R} = -7''.65$ $\qquad$ $d\text{R} = -0''.40$

$\text{K} = 4.4839$ $\qquad$ $d\mu = -0.090$

64 ξ *Cygni.*

Twelve observations (1838. 767) give,

$$\delta = +43^\circ\ 15'\ 11''.98.$$

Precession $= +14''.104$; sec var. $= +0''.219$; proper motion $= +0''.033$.

DATE.		E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1838,	March 8.	36.8	39.4	41.1	30.170	82° 15′. 02	417.67	— 3.05
„	„ 17.	34.6	39.6	40.3	29.377	82 15 18	409.25	— 2.41
„	„ 23.	30.4	35.0	36.6	29.549	82 15 13	413.15	— 3.52
„	„ 29.	43.5	46.6	48.1	30.408	82 15 10	416.00	— 1.84
„	April 8.	42.3	45.1	46	29.460	82 15 32	403.74	— 2.09
1839,	Feb. 20.	28.5	..	33.1	30.060	82 14 52	427.84	+ 1.34
„	„ 24.	33.1	..	37.2	29.467	82 14 75	415.18	+ 1.16
„	March 3.	39.3	..	43.5	29.820	82 14 81	413.24	— 0.09
„	„ 17.	34.9	..	39.5	29.917	82 14 79	417.30	— 1.41
„	„ 27.	40.8	..	45	29.070	82 15 10	398.47	— 3.27
„	April 6.	37.4	42.1	44.2	30.125	82 14 79	418.96	— 0.24
„	„ 7.	37.9	40.7	42	30.089	82 14 86	414.90	— 3.51
„	„ 11.	39.3	43	45.2	30.440	82 14 72	423.71	+ 1.89
„	„ 12.	43.3	45.8	46.9	30.270	82 14 92	411.74	— 4.26

$14 \times d\text{R} = -21''.30.$ $\qquad$ $d\text{R} = -1''.52.$

$\text{K} = 5.6710.$ $\qquad$ $d\mu = -0.268.$

2 D 2

17 *Andromedæ.*

Fifteen observations (1838. 801) give,

$\delta = + 42° \; 19' \; 39''. 41.$

Precession $= + 19''. 883$; sec var. $= + 0.051$; proper motion $= + 0.042$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, April 16.	31.3	35	35.9	29 578	83° 9′. 22	468.26	+ 1.25
,, ,, 17.	36.8	40.7	42.1	29 875	83 9 23	466 92	+ 2.45
,, ,, 22.	40	42.8	43	29 640	83 9 33	459.87	+ 0.81
,, May 3.	45.2	49.7	50	29 673	83 9 46	452.95	— 1.35
1838, May 5.	49.8	53	54	30.190	83 9 05	455.77	— 1.44
,, ,, 6.	52.8	54.3	55.9	30 156	83 9 09	453 39	— 0.36
,, ,, 8.	58.2	60	62.9	30.180	83 9 13	450 20	+ 1.43
1839, April 17.	37.9	42.8	43.4	29 101	83 8 78	448 57	— 3.56
,, ,, 18.	38.1	42.8	44 5	29.209	83 8 74	451.33	— 2.23
,, ,, 19.	40.2	42	43.8	29 764	83 8 62	458 92	— 1.13
,, ,, 24.	44.9	47.3	49.7	29.916	83 8 59	460.91	+ 2.43
,, May 2.	47	50.2	53.1	29 894	83 8 73	453.09	— 2.04
,, ,, 5.	47	49.7	51.1	29.786	83 8 75	451.66	— 1.91
,, ,, 7.	50.8	52.2	54.2	29.873	83 8 78	450.22	— 0.94

$14 \times d\text{R} = - 6''.59$ $\qquad d\text{R} = - 0''.47$

$\kappa = 6.2444$ $\qquad d\mu = - 0.075$

10 *Ursæ Majoris.*

Twelve observations (1837. 932) give,

$^{*}\delta = + 42° \; 26' \; 58''. 89.$

Precession $= - 13''. 522$; sec var. $= - 0''. 418$; proper motion $= - 0''. 294$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1835, Aug. 30.	53.9	58.5	..	29.870	83° 5′. 56	444.58	— 0.35
,, Sept. 6.	53.9	56.7	..	29.827	83 5 53	447.82	+ 2 47
,, ,, 8.	46.9	54.7	..	29.509	83 5 48	451.05	+ 5.00
,, ,, 12.	49	53.8	..	29.277	83 5 78	435.41	— 5.45

* Argelander's $\delta = 57''.80$; proper motion $= - 0''. 286$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.			OBS. REFRACT.	dR.
1835,	Sept.	15.	48.3	52 5	..	29.427	83°	5′.	65	441.67	— 1.92
„	Oct.	3.	46.1	50.2	..	29.227	83	5	64	445.37	+ 1.51
1837,	Aug.	30.	47.2	51.8	54 7	29.252	83	6	06	442.15	— 0.31
1838,	Sept.	9.	44.3	48.8	52.5	30 127	83	5	96	458.33	— 0.28
„	„	20.	44.5	..	53.9	29.606	83	6	27	446 88	— 3.86
„	„	23.	49.9	..	55 5	29.560	83	6	36	442.07	— 2.77
„	„	24.	46.8	..	56.9	29.721	83	6	21	451.16	+ 1.14
„	„	25.	45.5	..	54	29.860	83	6	17	453.78	— 0.78
„	Oct.	4.	45 1	..	55	30.286	83	6	09	460 06	— 0.24
1839,	Sept.	5.	52.9	57	57.9	29.474	83	6	69	434.36	— 5.60
„	„	10.	51.3	54.1	56.2	29 888	83	6	54	444 38	— 4.27
„	„	11.	48 4	52 2	55 1	29.714	83	6	56	443.35	— 5.44
„	„	21.	46 6	51.7	53.5	29.390	83	6	59	444 18	— 1.08
„	Oct.	2.	43.1	49.7	52	29 620	83	6	49	452.28	— 0.28
„	„	4.	42.1	45.1	47	29.888	83	6	36	460.33	+ 2.64
„	„	12.	46 8	48	50.1	29.664	83	6	57	448.62	— 1.17
„	„	16.	44.2	47.3	47 9	29.582	83	6	59	448 01	— 3.14
„	„	17.	41.2	49	49 9	29.956	83	6	51	445.46	— 4.19
„	„	18.	43.1	46.8	48.8	29.788	83	6	50	453.58	— 1.68
„	„	20.	45.9	48.8	49.4	29.778	83	6	57	450.11	— 2.32

$24 \times d\text{R} = -32''.38$ $\qquad$ $d\text{R} = -1''.35$

$\text{K} = 6.1247$ $\qquad$ $d\mu = -0''.220$

μ Ursæ Majoris.

Ten observations (1838. 235) give,

$$\delta = +42^\circ 21' 4''.05.$$

Precession $= -17''.877$; sec var. $= -0''.236$; proper motion $= -0''.015$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.			OBS. REFRACT.	dR.
1835,	Sept.	22.	49.9	52.9	..	28 907	83°	11′.	96	439.21	— 1.13
„	„	23.	47.3	52.4	..	29.285	83	11	90	443.02	— 5.51
„	„	24.	45	49 6	..	29 727	83	11	66	457.80	+ 0.16
„	Nov.	22.	39 3	45.6	..	29 411	83	11	86	460.16	+ 1.66
1838,	Sept.	23.	49.8	..	54 8	29.571	83	12	77	446.78	— 4.61
1839,	Sept.	30.	44	50.3	52.3	29.828	83	12	97	458 09	— 3.23
„	Oct.	2.	42 5	47.1	49.8	29 625	83	12	98	458.14	— 3.86
„	„	4.	42.9	45.8	47	29.919	83	12	88	464.49	+ 0.59

DATE.		E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1839,	Oct. 5.	41.3	47	50.2	30.148	83° 12′. 84	465.82	— 3.09
,,	,, 12.	45.5	47.5	49.2	29.703	83 12 97	461.13	+ 3.15
,,	,, 15.	43.2	48.9	49.1	29.570	83 13 04	457.58	— 0.66
,,	,, 16.	44.2	47.8	47.8	29.610	83 13 08	455.33	— 2.65
,,	,, 17.	39.9	46.1	47.5	29.947	83 12 91	465.57	— 1.75
,,	,, 20.	44.2	46.8	48.5	29.786	83 13 09	455.46	— 5.23
,,	,, 27.	41	46.1	47.3	30.298	83 12 86	470.96	— 0.75
,,	Nov. 11.	43.3	45.5	48.1	29.000	83 13 28	448.78	— 0.75
,,	,, 12.	41.9	45	47	29.320	83 13 24	452.70	— 3.19
,,	,, 13.	38.2	42.2	45.5	29.679	83 13 12	459.66	— 5.50

$18 \times d\text{R} = -36''.35$ $\qquad$ $d\text{R} = -2''.02$

$\text{K} = 6.2821$ $\qquad$ $d\mu = -0.321$

ν Persei.

Twelve observations (1838. 416) give,

$$\delta = +42° \, 2' \, 2''.57.$$

Precession $= +11.954$; sec var. $= -0.471$; proper motion $= -0.004$.

DATE.		E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	June 3.	45.2	..	54.6	29.891	83° 27′. 37	475.51	— 0.42
,,	,, 5.	50	56.9	57.3	30.005	83 27 35	476.63	+ 3.92
,,	,, 13.	52	55.1	57.1	29.500	83 27 66	458.63	— 4.40
,,	,, 14.	52.1	57.1	59	29.735	83 27 57	464.67	— 1.85
,,	,, 23.	62.4	63.8	65.3	30.122	83 27 68	457.66	— 4.70
1838,	June 12.	52	54.9	59	29.632	83 27 32	460.78	— 3.94
1839,	June 16.	52.9	57.9	58.8	30.144	83 27 05	468.37	— 3.25

$7 \times d\text{R} = -14''.64$ $\qquad$ $d\text{R} = -2''.09$

$\text{K} = 6.5578$ $\qquad$ $d\mu = -0.326$

58 *Aurigæ*.

Twelve observations (1837. 561) give,

$\delta = + 41° 58' 16''.86.$

Precession $= - 3''.376$; sec var. $= - 0''.613$; proper motion $= - 0''.138$.

DATE.		E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1833,	Aug. 14.	47.7	53.3	..	29.708	83° 32'. 89	473.59	— 2.77
1835,	July 29.	53 2	58	..	30.066	83 32 84	475.56	— 0.72
”	” 31.	56.5	62	..	29.990	83 32 87	473.71	+ 1.98
”	Aug. 30.	55.2	59.5	..	29.868	83 32 86	467.04	— 4.14
1837,	July 16.	54.2	59.1	61.2	29.897	83 32 96	470.52	— 2.03
”	” 20.	53.7	59	60.9	29.944	83 32 92	473.26	— 0.11
”	Aug. 5.	46.3	51	53.9	30.152	83 32 72	486.84	+ 2.05
”	” 6.	51	53.9	55	30.245	83 32 86	478.50	— 3.03
”	” 7.	49.4	54.8	58	30.260	83 32 77	483.46	+ 0.27
”	” 15.	57.9	61	63	30.084	83 32 99	470.93	— 0.87
”	” 26.	49	54.7	56.5	29.939	83 32 94	475.06	— 3.63
”	” 29.	46.5	52	54.6	29.429	83 32 99	471.83	— 1.31
1838,	Aug. 4.	54.8	..	60	29.204	83 33 18	461.39	+ 0.21
”	” 11.	56.9	..	62.2	29.764	83 33 09	467.64	— 0.22
”	” 12.	56.3	..	61.8	29.840	83 33 07	477.92	— 0.99
”	” 13.	51.8	..	58.5	30.060	83 32 92	467.64	+ 0.08
1839,	July 15.	50.1	52.8	57.3	29.853	83 33 00	474.37	— 1.85
”	” 19.	51.4	54.4	59.2	29.071	83 33 20	462.91	+ 0.45
”	” 24.	52.9	58	59.3	29.578	83 33 15	466.32	— 2.82
”	” 27.	52.2	60	61.5	29.636	83 33 07	471.34	+ 0.84
”	” 31.	47.8	52	55.2	29.624	83 33 05	472.63	— 2.37
”	Aug. 2.	56.1	57.1	59.8	29.762	83 33 15	467.52	— 1.25
”	” 4.	52.3	57.4	59.9	30.184	83 32 98	477.59	— 1.68
”	” 12.	49.1	56	58	30.124	83 32 94	480.66	— 0.84

$24 \times d\text{R} = - 24''.75$ $\qquad$ $d\text{R} = - 1''.03$

$\kappa = 6.5578$ $\qquad$ $d\mu = - 0.157$

γ Andromedæ.

Twelve observations (1837. 531) give,

*δ = + 41° 30′ 34″. 54.

Precession = + 17″.647; sec var. = — 0″. 260; proper motion = — 0″.057.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.			OBS. REFRACT.	dR.
1836,	May	28.	54.2	59.4	61	30.281	83°	58′.	12	506.19	— 1.47
1837,	„	12.	46.7	51.6	52.1	29.517	83	57	88	499.84	— 2.78
„	„	14.	44 5	51.8	52.8	30.013	83	57	66	513.25	— 0.08
„	„	18.	44.8	48.4	50	30.193	83	57	61	516.38	+ 0.24
„	„	26.	43.5	51.1	51	29.588	83	57	72	510.12	+ 2.89
„	„	27.	50	54.2	54.1	29.800	83	57	84	503.23	— 0.52
„	„	30.	46.9	52 5	53.2	29.837	83	57	74	508.82	+ 1.15
„	June	3.	48.7	54.6	56.8	29.896	83	57	77	506.96	+ 0.40
1838,	May	15.	38.2	44	46.9	29 684	83	57	33	512.64	— 1.91
„	„	17.	39	45.9	47.1	29.716	83	57	32	513 95	— 0.20
„	„	23.	47.4	51.3	53.3	29.785	83	57	54	500.85	— 4.08
„	„	24.	48.3	52.1	53.9	29.870	83	57	42	508.03	+ 0.84
„	„	25.	50 4	54	55.5	29.906	83	57	47	505.28	+ 0.70
„	„	26.	52	54.8	55.9	29.931	83	57	45	506.31	+ 3.17
1839,	May	25.	46.7	51	53.7	30.208	83	57	05	510.21	— 3.21
„	„	26.	46.2	50	53.1	29.988	83	57	13	505.87	— 4.06
„	„	28.	56.2	57.1	58	30.064	83	57	24	499.33	— 3.75
„	„	29.	53 8	56.2	60	30 077	83	57	16	503.79	+ 0.56
„	„	30.	56.1	58	61.2	30.044	83	57	22	500.25	+ 0.02
„	„	31.	57	58.8	62.1	29 916	83	57	27	497.58	— 0.67
„	June	1.	52.8	55.5	59.2	29.786	83	57	21	500.65	+ 1.18
„	„	2.	52.1	56	59.8	29.624	83	57	26	498.09	+ 0.58
„	„	3.	46.9	50.4	52.5	29.500	83	57	20	501.67	+ 0.41

$23 \times d\text{R} = - 10''.59$ $\qquad$ $d\text{R} = - 0''.46$

$\kappa = 7.1337$ $\qquad$ $d\mu = - 0''.065$

* Argelander's δ . . . = 35″. 20 Mine, . . . 34″. 74
Airy, Greenwich, (1836 and 1837,) 34 11

58 *Persei.*

Eight observations (1837. 198) give,

$\delta = + 40° 54' 24''. 32.$

Precession $= + 8''.071$; sec var. $= - 0''.329$; proper motion $= - 0''.035$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, June 11.	50.7	58	58.7	29.506	84° 34'. 35	540.79	— 3.89
„ „ 13.	51.7	55.5	57	29.502	84 34 27	546.15	+ 2.77
„ „ 14.	51.2	55.1	57.2	29.735	84 34 26	547.53	— 0.78
1839, „ 16.	51.6	56.8	58.8	30.144	84 33 87	551.15	— 3.66
„ „ 28.	47.1	50	53.8	29.881	84 33 89	550.96	— 4.89
„ „ 29.	45.9	50.1	54.9	30.102	84 33 72	560.81	— 0.28

$6 \times d\text{R} = - 10''.73$ $\qquad$ $d\text{R} = - 1''.79$

$\text{K} = 7.8566$ $\qquad$ $d\mu = - 0.228$

58 *Cygni.*

Twelve observations (1838. 024) give,

$\delta = + 40° 30' 58''. 86.$

Precession $= + 13''. 603$; sec var. $= + 0''. 233$; proper motion $= + 0''. 018$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, March 24.	28.5	33.9	35.1	29.722	84° 56'. 49	604.06	— 9.41
„ „ 29.	32.2	36.1	38.2	29.530	84 56 58	599.77	— 4.47
„ April 1.	33.9	37.0	39	29.812	84 56 40	610.71	+ 2.96
1838, March 8.	36.8	39.4	41	30.170	84 56 20	604.29	— 5.79
„ „ 17.	34.6	39.6	40.3	29.377	84 56 38	595.87	— 1.39
„ „ 23.	30.4	35	36.6	29.459	84 56 32	600.31	— 4.61
„ „ 29.	43.5	46.6	48.1	30.408	84 56 35	599.37	— 6.09
1839, April 6.	37.8	42.2	44.2	30.125	84 55 98	606.53	— 0.66
1840, Feb. 26.	31.8	35.8	37	30.357	84 55 43	619.21	— 0.74
„ „ 27.	28.5	34	35.3	30.258	84 55 41	620.33	— 2.47
„ „ 29.	32	35.7	36.5	30.264	84 55 48	615.43	— 2.48
„ March 1.	30.8	33.6	34.9	30.330	84 55 43	618.21	— 2.80
„ „ 2.	34.5	35.7	36.2	30.382	84 55 59	609.34	— 7.60

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1840, March 3.	34.2	35.8	37.2	30.415	84° 55′. 40	621.50	+ 3.82
" " 4.	35 5	37.2	38.2	30.247	84 55 45	618.75	+ 6.38
" " 5.	38.2	38.2	40.2	30.108	84 55 71	603.40	— 2.67
" " 6.	44.2	43.1	43.1	30.249	84 55 84	595.56	— 5.10
" " 9.	40.3	42.8	43.5	30.481	84 55 69	605.34	— 5.13
" " 18.	38.2	42.8	44 5	30.150	84 55 79	600.66	— 3.91
" " 20.	35.6	40.1	43.2	30.380	84 55 75	603.22	— 9.06
" " 23.	36	37.9	40.2	30.261	84 55 71	606.31	— 5.98

$21 \times d\text{R} = -67''.20 \qquad d\text{R} = -3''.20$

$\text{K} = 8.8831 \qquad d\mu = -0.360$

The discordances in the separate values of $d\mu$ have obviously no relation to the zenith distance, or the time of year, and may therefore be regarded as casual.

If we combine them according to the method already assigned, we obtain,

NAME.	NO. OBS.	$nd\text{R} \times \text{K}$	$n \times \text{K}^2$.	$d\mu$.
45 Cygni.	17	— 81.5223	445.642	— 0″.576
31 "	16	— 31.2112	224.400	— 0 160
Capella.	20	— 59.1490	278.526	— 0 212
Pxxi. 157.	8	+ 2.4732	131.505	+ 0 019
22 Andromedæ.	17	— 102.0713	293.630	— 0 348
β Aurigæ.	30	— 128 6188	530.360	— 0 242
α Cygni.	39	— 269.4501	813.976	— 0 331
46 Andromedæ.	19	— 34.3018	382.002	— 0 090
64 Cygni.	14	— 120.7923	450.243	— 0 268
10 Ursæ Majoris.	24	— 198.2566	900.287	— 0 220
17 Andromedæ.	14	— 41.1506	545.895	— 0 075
μ Ursæ Majoris.	18	— 228.3543	710.366	— 0 321
ν Persei.	7	— 93.8717	287.796	— 0 326
58 Aurigæ.	24	— 162.3056	1032.114	— 0 157
γ Andromedæ.	23	— 75.5459	1170.462	— 0 065
58 Persei.	6	— 84.3546	370.351	— 0 228
58 Cygni.	21	— 596.9444	1657.992	— 0 360
Sum . .	317	— 2305.4273	10225.547	

Hence

$$d\mu = -\frac{2305.4273}{10225.547} = -0.2255.$$

The value of μ used in computing the refractions is,

$$\begin{aligned} \mu &= 57.7682\,; \\ d\mu &= -0.2255\,; \\ \text{sum} &= \overline{57.5427.} \end{aligned}$$

This may perhaps require a correction for the run of the microscopes, which though very small is sensible. From the erection of the circle to July 8, 1837, its effect on the mean of four microscopes was $= -\dfrac{0''.\,18 \times \text{A}'}{5'\;0''}$. At this time it was changed by the rough operations necessary in attaching another pair of microscopes, and has been since considered permanent at $+\,0''.\,41 \times \dfrac{\text{A}'}{5'}$. This is, however, a mean value, being deduced from readings of the four, in 30 equidistant positions of the circle. Hence I found as above

$$d^2\mu = +\frac{38''.2909}{10225.547} = +\,0''.0037$$

and

$$\mu = 57''.5464$$

a value whose near approximation to BESSEL's 57''.524, will prove very remarkable, if when I have means of determining the length of the seconds' pendulum here, it should be found little different from that of Königsberg. That observatory is a little north of me, but it is only 90 feet above the Baltic; while this is 211 feet above the sea, and the substratum, dense limestone, so that the local gravity must be nearly alike in both cases.

As to the southern stars, I have used the declinations of the St. Helena catalogue, reduced to BESSEL's refractions, by the table given page 22, and those of Professor HENDERSON. (Mem. R. Ast. Soc. X. 80.) The two are not strictly comparable in respect of refraction, for the St. Helena Observatory, being 700 feet above the sea, and resting on dense volcanic rocks, may be expected to have an excess of gravity above the Cape, and therefore larger refraction. At the latter place I find, by comparing the length of the pendulum with that of Greenwich, that BESSEL's refractions should be multiplied by 0.9984; and, in fact, HENDERSON's observations on refraction shew, that even a greater diminution is required. I have not, however, changed them further than

by reducing them to 1830, with the precession, &c., annexed to each star. When possible, the proper motions are deduced by comparison with Airy's Greenwich places.

24. o^2 *Canis Majoris.*

$\delta = -23° 35' 23''.83$. J. (Johnson).

Precession $= -4''.846$; sec var. $= -0''.352$; proper motion $= +0''.011$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	Feb.	18.	34.3	41	41	29.274	77° 52'. 73	269.48	+ 2.37
„	March	12.	29.8	35.6	36.8	29.575	77 52 85	274.78	+ 1.19
„	„	13.	30.7	34.8	36	30.174	77 52 66	276.29	— 2.10
„	„	17.	38.6	41.8	43	30.211	77 52 74	271.47	— 2.76
„	„	21.	38.8	40.1	41.1	29.712	77 52 79	268.76	— 0.88
„	„	23.	32.2	36	37.6	29 663	77 52 76	270.51	— 3.10
„	„	24.	32	36.5	38.1	29.727	77 52 77	270.15	— 3.51
1838,	Feb.	8.	38.7	39.8	40.2	28.524	77 53 02	254.86	— 4.14
„	„	13.	27	30	31.2	29.479	77 52 86	270.93	— 3.51
„	„	20.	31.8	34.7	35.7	29.483	77 52 76	273.16	+ 1.59
„	„	21.	32.9	36.1	38	29.583	77 52 82	269.90	— 2.06
„	March	15.	39.2	44.8	47.2	29.798	77 52 88	267.99	— 2.08
„	„	17.	36.2	39.8	41.2	29.344	77 52 93	265.12	— 2.64

$13 \times d\text{R} = -21''.63$ $\qquad$ $d\text{R} = -1''.16$

$\text{K} = 5.2240$ $\qquad$ $d\mu = -0.318$

15 *Argûs.*

* $\delta = -23° 49' 8''.58$. (J. H.)

Precession $= -10''.051$; sec var. $= -0''.317$; proper motion $= +0''.075$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	March	13.	29.2	34.1	35	30.185	78° 6'. 92	282.69	— 2.54
„	„	14.	34.1	37.3	38.4	30.287	78 6 96	280.78	— 2.28
„	„	23.	32.2	34.5	35.8	29.657	78 7 03	277.17	— 1.17

* Johnson's δ . . . $= 7''.80$ $\qquad$ Henderson's δ . . $= 9''.36$

Had the first been used, the refractions would be $0''.78$ less; $d\mu = -0''.306$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	April	3.	35	37.8	39	29.429	78° 7′. 11	272.91	— 1.63
„	„	4.	35.7	38.7	40.3	29.683	78 7 00	279.65	+ 3.22
1838,	Feb.	20.	31.2	34.4	35	29.496	78 7 08	279.34	+ 2.90
„	„	21.	31.8	35	36.9	29.577	78 7 13	276.93	— 0.96
„	March	17.	35.8	39.1	40.9	29.368	78 7 22	274.61	+ 1.11
1839,	Feb.	20.	29.6	..	34.1	30.066	78 7 20	283.91	+ 0.06
„	„	24.	33.8	..	38	29.461	78 7 13	274.69	— 0.86
„	March	17.	35	..	40	29.907	78 7 12	279.87	+ 0.89
„	„	25.	37.9	..	43.9	29.424	78 7 59	269.77	— 3.14
„	April	5.	40.4	..	44	29.722	78 7 47	272.52	— 1.69
„	„	6.	39	44	45.8	30.118	78 7 39	273.57	— 1.05
„	„	7.	38.5	40.2	43.2	30.094	78 7 45	276.75	— 2.00
„	„	11.	41.8	45.2	47	30.442	78 7 42	274.51	— 5.48

$16 \times d\text{R} = -14''.62$ $\qquad d\text{R} = -0''.91$

$\text{K} = 5.5356$ $\qquad d\mu = -0.165$

16. o^1 *Canis Majoris.*

$\delta = -23° \; 58' \; 35''.82$. J.

Precession $= -4''.092$; sec var. $= -0''.353$; proper motion $= -0''.059$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	Feb.	18.	34.3	41	41	29.274	78° 15′. 70	279.53	+ 3.05
„	March	12.	29.8	35.6	36.8	29.575	78 15 68	283.13	+ 1.06
„	„	13.	30.7	34.8	36	30.174	78 15 71	281.70	— 5.65
„	„	17.	38.6	41.8	43	30.211	78 15 76	278 48	— 3.56
„	„	23.	32.2	36	37.6	29.663	78 15 76	279.06	— 2.43
„	„	24.	32	36.5	38.1	29.727	78 15 73	280.69	— 1.62
1838,	Feb.	8.	38.7	39.8	40.2	28.524	78 15 97	265.28	— 1.72
„	„	13.	27	30	31.2	29.479	78 15 72	281.30	— 1.67
„	„	20.	31.8	34.7	35.7	29.483	78 15 72	282.64	+ 2.59
„	„	21.	32.9	36.1	38	29.583	78 15 79	278.13	— 1.89
„	March	15.	39.2	44.8	47.2	29.798	78 15 87	275.23	— 3.36
„	„	17.	36.2	39.8	41.2	29.344	78 15 91	273.32	— 4.36
„	„	23.	33.5	35.2	39.7	29.500	78 15 87	275.28	— 3.91

$13 \times d\text{R} = -23''.47$ $\qquad d\text{R} = -1''.81$

$\text{K} = 5.5514$ $\qquad d\mu = -0.325$

ξ Argus.

*$\delta = -24° 26' 17''.90$. (J.)

Precession $= -8''.647$; sec. var. $= -0''.329$; proper motion $= -0''.012$. (A.)

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, March 12.	28.5	33.4	35	29.604	78° 43′. 68	296.33	+ 1.38
„ „ 13.	29.2	34.1	35	30.182	78 43 62	300.13	— 0.04
„ „ 14.	34.2	37.6	39	30.287	78 43 68	296.36	— 1.64
„ „ 29.	32.2	38.6	40	29.521	78 43 59	290.11	— 1.47
„ „ 30.	36.1	38.2	42	29.757	78 43 61	288.93	— 2.57
1838, Feb. 20.	31.4	34	35.2	29.496	78 43 81	293.98	+ 1.93
„ „ 21.	31.9	35.2	36.9	29.577	78 43 87	290.84	— 1.68
„ March 29.	45.1	47.1	48.5	30.410	78 43 95	289.59	— 2.93
1839, Feb. 20.	29.6	..	34.1	30.066	78 43 86	297.64	— 1.18
„ March 17.	35.1	..	40.1	29.912	78 44 02	291.56	— 2.24
„ „ 25.	38.9	..	44.1	29.424	78 44 15	283.90	— 2.83
„ April 5.	40.3	..	44	29.717	78 44 16	284.37	— 4.48
„ „ 7.	38.9	40.4	43.2	30.094	78 44 07	289.26	— 4.12

$13 \times d\text{R} = -21''.87$ $\qquad$ $d\text{R} = -1''.68$

$\kappa = 5.7931$ $\qquad$ $d\mu = -0.290$

22 λ *Sagittarii.*

†$\delta = -25° 30' 23''.90$. (J.)

Precession $= +1''.528$; sec. var. $= +0''.537$; proper motion $= -0''.291$. (J.)

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, July 20.	54.7	57.3	61	29.940	79° 46′. 40	308.74	— 1.29
„ „ 27.	55	57	61.5	29.571	79 46 48	306.47	+ 0.43
„ August 5.	46.9	51	53.9	30.152	79 46 36	313.99	+ 0.53

* Airy (15 observations, 1836-7) . 18″.93

† The declinations of this star are discordant:

Airy (16 in 1837) . . . . . 25″.79 $\qquad$ Maclear . . . . . 24″.45

Johnson „ . . . . 23 .90

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	Aug.	6.	51.2	54	56.2	30.240	79° 46′. 32	316.17	+ 0.76
,,	,,	7.	50.3	54	58	30 261	79 46 20	313.75	— 2.54
,,	,,	14.	56.8	60	62	30.070	79 46 43	309.84	+ 0.05
,,	,,	15.	58	61	63	30.082	79 46 45	308.51	— 0.84
,,	,,	16.	60.6	62.1	64	29.968	79 46 54	305.71	— 0.95
,,	,,	29.	47.6	52.2	56	29.430	79 46 41	311.51	+ 2.19
,,	,,	31.	48.7	53	56.1	29.275	79 46 49	306.92	— 0.16
1839,	July	15.	50.2	52.8	57.3	29.853	79 46 29	315.08	+ 3.41
,,	,,	19.	51.8	54 5	59.2	29.071	79 46 51	301.84	— 0.68
,,	,,	24.	53.7	59.2	61	29.578	79 46 42	307.17	+ 0.62
,,	,,	28.	52.4	57.2	61.8	29.778	79 46 13	311.02	+ 2.96
,,	,,	31.	48.1	53.8	56.1	29.622	79 46 32	313.22	+ 2.62
,,	Aug.	2.	56.9	57.9	60.1	29.764	79 46 46	305.70	— 0.84
,,	,,	4.	52.9	58.1	60.2	30.186	79 46 30	314.58	+ 0.44
,,	,,	19.	47.1	54.2	56.2	29.960	79 46 29	315.52	+ 0.72
,,	,,	20.	50.8	56	57.8	30.084	79 46 33	313.05	— 0.66
,,	,,	21.	53.2	56.2	58.9	29.932	79 46 36	311.48	+ 0.90
,,	,,	26.	51.2	55.5	59.1	29.620	79 46 39	310.31	+ 1.73
,,	Sept.	5.	55.9	58	61.7	29.428	79 46 50	303.39	— 0.28
,,	,,	11.	51.2	57.2	60	29.736	79 46 42	308.58	— 1.20

$23 \times d\text{R} = + 7''.92$ $\qquad$ $d\text{R} = + 0''.34$

$\kappa = 6.1035$ $\qquad$ $d\mu = + 0.056$

Antares.

* $\delta = - 26° 2' 47''.69$. (J. and H.)

Precession $= - 8''.556$; sec var. $= + 0''.487$; proper motion $= - 0''.031$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	June	14.	51.2	55.1	57.2	29.735	80° 19′. 80	325.90	— 0.49
,,	,,	15.	50.9	57.1	61.1	30.090	80 19 72	331.25	+ 0.43
,,	July	7.	56.2	59	62.2	30.106	80 19 83	325.18	— 2.27
,,	,,	9.	56.9	63.7	65.4	29.905	80 19 89	321.55	— 3.16
,,	,,	10.	60.1	64.2	66.5	29.846	80 19 93	319.50	— 2.47
,,	,,	16.	56.6	61	63.1	29.923	80 19 85	322.83	— 2.37
,,	,,	18.	57	60.2	63.5	29.612	80 19 81	320.07	— 1.40

* Airy, δ (18 obs. in 36 and 37) .	48″.11	Argelander	46″.50
Henderson	48 .68	Mine	47 .44
Johnson, ...	46 .71		

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, July 20.	57.0	60	63	29.934	80° 19′. 82	325.53	+ 0.55
„ August 5.	51.2	53.8	57.1	30.147	80 19 76	329.79	— 1.56
„ „ 15.	63	63	66	30.075	80 19 97	320.11	— 2.53
1838, July 1.	53.2	57.3	58.7	29.806	80 19 99	325.10	— 1.23
„ „ 25.	53.2	57.5	60	29.904	80 20 00	325.21	— 2.15
„ „ 31.	57.3	..	62	29.812	80 20 01	324.79	+ 1.32
„ August 4.	58.8	..	63	29.192	80 20 14	313 62	— 2.35
1839, June 14.	48.1	53.8	56.2	29.892	80 20 03	331.01	+ 0.21
„ „ 16.	51.6	56.8	58.8	30.144	80 20 06	329.49	— 1.70
„ „ 28.	47.1	50	53.8	29.881	80 20 03	331.33	+ 0.66
„ „ 29.	45.9	50.1	54.9	30.102	80 19 94	337.18	+ 2.54
„ July 9.	52	54.8	58.8	29.370	80 20 26	318.29	— 4.17
„ „ 10.	53.9	56.6	60.2	29.517	80 20 22	320.71	— 2.00
„ „ 20.	55.3	57.2	60	29.360	80 20 23	320.04	— 0.07
„ „ 22.	55.4	59	60.9	29.750	80 20 15	324.57	— 0.78

$22 \times d\text{R} = -24''.99$ $\qquad$ $d\text{R} = -1''.14$

$\kappa = 6.3200$ $\qquad$ $d\mu = -0.180$

19 δ *Sagittarii.*

$\delta = -29^\circ\ 53'\ 25''.75$ (J).

Precession $= +0''.884$; sec var. $= +0''.559$; proper motion $= -0''.014$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, July 20.	54.8	57.3	62	29.938	84° 6′. 24	507.06	+ 0.36
„ „ 27.	55	57	61.5	29.571	84 6 37	499.91	— 4.38
„ Aug. 6.	51.2	54	56.2	30.240	84 6 10	516.16	— 3.91
„ „ 7.	50.3	54	58	30.261	84 5 95	524.71	+ 3.45
„ „ 14.	56.8	60	63	30.069	84 6 34	508.72	— 8.85
„ „ 15.	58.6	61	63	30.082	84 6 30	504.49	— 6.74
„ „ 16.	60.6	62.1	64.5	29.970	84 6 33	502.80	— 2.47
„ „ 29.	47.6	52.2	56	29.430	84 6 16	513.58	+ 3.08
„ „ 31.	48.7	53	56.1	29.275	84 6 27	506.99	+ 0.78
1838, Aug. 4.	55.1	..	60.5	29.200	84 6 45	494.66	— 2.04
„ „ 14.	52.1	..	60	30.040	84 6 18	511.22	— 4.24
1839, July 15.	50.3	52.8	57.3	29.853	84 6 13	511.52	— 2.81
„ „ 24.	53.7	59.2	61	29.578	84 6 17	509.15	+ 0.98
„ „ 31.	48.1	53.8	56.1	29.622	84 6 17	509.50	— 2.34
„ Aug. 11.	50.9	56	58.9	30.162	84 6 07	516.65	— 2.19
„ „ 19.	47.1	54.2	56.2	29.960	84 6 08	516.41	— 3.34
„ Sept. 5.	55.9	58	61.7	29.428	84 6 32	501.96	+ 1.11
„ „ 11.	51.2	57.2	60	29.736	84 6 29	504.57	— 7.76

$18 \times d\text{R} = -41''.41$ $\qquad$ $d\text{R} = -2''.30$

$\kappa = 9.5710$ $\qquad$ $d\mu = -0.241$

34 σ *Sagittarii.*

*$\delta = -26^\circ\ 29'\ 55''.31$. (J.)

Precession $= +3''.889$; sec var. $= +0''.532$; proper motion $= -0''.093$.

DATE.		E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1838,	Aug. 4.	54.8	..	60	29.204	80° 45′. 20	330.87	— 1.19
,,	,, 13.	51.8	..	58.5	30.060	80 45 09	309.30	— 4.91
,,	,, 14.	52.9	..	58.2	30.033	80 45 06	340.34	— 2.80
1839,	July 19.	51.4	54	57.3	29.072	80 45 09	333.01	+ 0.37
,,	,, 24.	53.7	59.2	61	29.578	80 45 07	334.58	— 5.18
,,	,, 28.	51.5	55.7	60	29.776	80 44 97	340.62	— 0.43
,,	,, 31.	47.4	51.2	55	29.627	80 44 93	342.88	+ 0.46
,,	Aug. 2.	56.1	57.1	59.8	29.762	80 45 06	335.22	— 2.56
,,	,, 3.	52.7	56.8	59.2	30.026	80 44 95	341.96	— 1.11
,,	,, 4.	52.3	57.4	59.2	30.184	80 44 90	345.06	— 0.12
,,	,, 11.	51	56.2	58.2	30.169	80 44 89	345.50	— 0.45
,,	,, 12.	49.1	56	58	30.124	80 44 82	350.24	+ 3.50
,,	,, 19.	47.7	51.7	56.2	29.960	80 44 92	344.39	— 1.56
,,	,, 21.	53.1	55.4	58.1	29.930	80 45 01	339.40	— 2.41
,,	,, 24.	54.8	57.7	60	29.745	80 45 00	339.45	+ 1.02
,,	,, 26.	51	54	59.1	29.620	80 44 98	341.03	+ 1.40
,,	Sept. 5.	54.9	57	60.8	29.442	80 45 10	334.20	— 0.73

$17 \times d\text{R} = -16''.70$ $\qquad$ $d\text{R} = -0''.98$

$\kappa = 6.7651$ $\qquad$ $d\mu = -0.145$

* This star is doubtful.

δ by Airy (3 observations), ...	57″.52
Henderson (Edinburgh, 5 obs.), Bessel's Refraction, .	54 .56
,, Cape, ...	58 .11
Maclear, Direct, ...	58 .17
,, Reflected, ...	57 .23
Johnson, ...	55 .31

ϵ *Canis.*

$^{*}\delta = -28° 44' 45''.35$ (J. and H.)

Precession $= -4''.507$; sec var. $= -0''.333$; proper motion $= -0''.011$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, Feb. 18.	34.8	39.6	40.2	29.295	82° 59'. 03	452.35	+ 3.28
„ March 12.	29.3	33.5	36.8	29.575	82 58 95	460.15	+ 1.31
„ „ 13.	30.4	34.7	36	30.177	82 58 89	463.88	— 3.05
„ „ 17.	38.2	41.4	43.1	30.211	82 59 01	453.46	— 6.04
„ „ 23.	32.1	35.2	37.6	29.663	82 59 00	457.54	+ 0.28
„ „ 24.	32	36.5	38.1	29.727	82 59 02	456.63	— 1.73
1838, Feb. 8.	38.7	39.5	40.2	28.524	82 59 43	430.66	— 3.38
„ „ 13.	27	30	31.2	29.478	82 59 02	456.53	— 3.29
„ „ 21.	32.9	36.1	38	29.583	82 59 14	450.18	— 5.18
1839, Feb. 12.	36.2	..	41.2	30.034	82 59 05	459.29	+ 0.50
„ „ 14.	36.1	..	40.9	29.734	82 59 13	454.90	+ 0.50
„ „ 17.	22.7	..	29.8	29.210	82 59 08	458.68	— 1.38
„ „ 18.	29.7	..	34.1	29.400	82 59 15	454.22	— 1.67
„ „ 20.	29.8	..	34.9	30.054	82 58 95	466.47	+ 0.75
„ March 3.	40.2	..	45.5	29.820	82 59 22	452.36	+ 0.74
„ „ 17.	35.2	..	40.5	29.912	82 59 15	457.61	— 0.49

$16 \times d\text{R} = -18''.85.$ $\qquad d\text{R} = -1''.18.$

$\kappa = 8.6376.$ $\qquad d\mu = -0.136.$

31 η *Canis Majoris.*

$^{\dagger}\delta = -28° 58' 35''.79$ (J.)

Precession $= -6''.642$; sec var. $= -0''.323$; proper motion $= -0''.011$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, Feb. 18.	34.8	39.6	40.2	29.295	83° 12'. 90	465.05	+ 2.24
„ March 14.	34.7	38.6	40.8	30.287	83 12 74	477.79	— 0.62
„ „ 17.	38.2	41.4	43.1	30.211	83 12 89	469.29	— 4.19
„ „ 23.	32.1	35.2	37.6	29.663	83 12 85	471.85	+ 0.67

* δ by Airy (26 obs.) . . 46''.38 Henderson, Cape, ... 46''.36

† Henderson's declination is a second greater, but rests on a much less number of observations.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
„ „ 24.	32	36.5	38.1	29.727	83 12 83	473 37	+ 1.12
„ „ 30.	37	40.1	42.1	29.756	83 13 00	463.15	— 4.39
1838, Feb. 8.	38.5	39.3	40.2	28.524	83 13 30	444.99	— 1 94
„ „ 21.	32.4	35.5	38	29.583	83 12 98	473.32	+ 3.62
1839, Feb. 9.	39	..	43.7	30 064	83 13 04	468.45	— 1.86
„ „ 12.	36.2	..	41	30 040	83 13 04	468.91	— 4.10
„ „ 14.	35.9	..	40.8	29 733	83 13 11	465.39	— 3.16
„ „ 17.	22.3	..	29.8	29.220	83 12 93	477.02	+ 2.32
„ „ 18.	30.7	..	34.1	29.394	83 13 09	467.04	— 1.65
„ „ 20.	29.9	..	34.3	30.058	83 12 92	477.85	— 2.20
„ March 17.	35.4	..	40.1	29.908	83 13 10	470.39	— 1.47
„ „ 25.	40.1	..	44.1	29.416	83 13 33	456.61	— 2.76

$16 \times d\text{R} = -18''.37$ $\qquad d\text{R} = -1''.15$

$\text{K} = 8.8592$ $\qquad d\mu = -0''.130$

δ Canis Majoris.

$\delta = -26^\circ\, 7'\, 42''.18.$ (J.)

Precession $= -5''.316$; sec var. $= -0''.340$; proper motion $= +0''.021$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, Feb. 18.	34.8	39.6	40.2	29.295	80° 24′. 02	335.17	— 0.90
„ March 12.	29.3	33.5	36.8	29.575	80 23 93	343.39	+ 0.18
„ „ 13.	30.4	34.7	36	30.177	80 23 88	346.83	— 2.54
„ „ 14.	34.7	38.6	40.8	30.287	80 23 95	343.81	— 3.55
„ „ 17.	38.2	41.4	43.1	30.211	80 23 99	340.16	— 3.81
„ „ 23.	32.1	35.2	37.6	29.663	80 23 94	343.68	+ 1.53
„ „ 24.	32	36.5	38.1	29.727	80 24 02	338.56	— 4.42
1838, Feb. 8.	38.5	39.3	40.2	28.524	80 24 28	321.45	— 3.17
„ „ 13.	27	30	31.2	29.479	80 24 00	339.75	— 4.16
„ „ 20.	31.8	34.7	35.7	29.483	80 24 17	338.76	— 1.70
„ „ 21.	32.9	36.1	38	29.583	80 24 03	339.11	— 1.57
„ March 15.	39.2	44.8	47.2	29.798	80 24 13	335.69	— 2.74
„ „ 17.	36.2	39.8	41.2	29 344	80 24 19	338.60	— 1.80
„ „ 23.	33.5	35.2	39.7	29.500	80 24 09	338.62	— 0.68

$14 \times d\text{R} = -29''.33$ $\qquad d\text{R} = -2''.09$

$\text{K} = 6.5921$ $\qquad d\mu = -0.318$

ζ Canis Majoris.

$\delta = -29° 59' 34''.62$ (J. H.)

Precession $= -1''.205$; sec var. $= -0''.335$; proper motion $= -0''.022$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, Feb. 18.	34.7	41.6	41.4	29.264	84° 12'. 13	533.37	+ 3.77
„ March 12.	30	36.4	37	29.562	84 12 09	537.47	— 3.17
1838, Feb. 8.	39	39.8	40.6	28.530	84 12 48	510.98	— 0.89
„ „ 13.	27.6	30.3	31.8	29.474	84 12 15	539.80	— 2.61
1839, Feb. 10.	42	..	44	30.116	84 12 23	527.50	— 9.27
„ „ 12.	36.2	..	41.4	30.034	84 12 02	540.77	— 1.01
„ „ 14.	36.3	..	41.5	29.735	84 12 13	534.72	— 2.63
1840, Feb. 13.	34.8	37	40.2	29.625	84 12 15	534.57	— 2.30
„ „ 26.	33.8	36.7	40.1	30.370	84 12 14	542.88	— 8.82
„ „ 28.	32.7	35	37.2	30.234	84 11 96	548.47	— 2.02
„ March 2.	33.5	35	38.5	30.386	84 11 89	553.44	+ 1.33
„ „ 3.	34.8	36.4	38.4	30.416	84 11 87	554.56	+ 3.43
„ „ 4.	35.8	38	40	30.254	84 12 09	541.50	— 5.59
„ „ 5.	38.2	39.7	41.5	30.128	84 12 13	539.00	— 2.90
„ „ 9.	44.9	44.8	45	30.477	84 12 11	540.30	+ 0.55
„ „ 17.	42.2	46	49.1	30.214	84 12 16	537.84	— 0.45
„ „ 18.	41	45.3	49	30.146	84 12 17	537.55	— 0.95

$17 \times d\text{R} = -33''.53$ $\qquad$ $d\text{R} = -1''.97$

$\kappa = 10.0672$ $\qquad$ $d\mu = -0''.196$

38. *ζ Sagittarii.*

* $\delta = -30° 6' 49''.15$. (J.)

Precession $= +4''.487$; sec var. $= +0''.543$; proper motion (J.) $= -0''.013$.

DATE.	E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837, Aug. 5.	46.1	51	53.7	30.155	84° 18' 64	538.27	— 2.57
„ „ 6.	48.8	53.2	55	30.248	84 18 48	546.92	+ 6.15
„ „ 7.	49.2	54.8	56.2	30.259	84 18 63	538.23	— 0.39

* The proper motion is deduced from J., as Airy's places for 1836 and 1837 differ 2''.68.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1837,	Aug.	15.	57.9	60.4	62.8	30.090	84° 18′. 97	518.33	— 8.92
„	„	16.	59.2	62	64	29.965	84 18 98	518.13	— 4.12
„	„	26.	49	54.7	56.5	29.939	84 18 77	531.06	— 3.10
„	„	29.	46.5	52	54.6	29.429	84 18 86	525.97	— 1.99
1838,	Aug.	4.	54.8	..	60	29.204	84 18 94	515.17	+ 1.34
„	„	13.	51.8	..	58.5	30.060	84 18 75	527.16	— 5.21
„	„	14.	53	..	58.2	30.033	84 18 70	530.00	— 0.42
1839,	July	24.	52.5	57.4	59.2	29.578	84 18 65	526.57	+ 3.79
„	„	27.	52.2	60	61.5	29.636	84 18 68	524.82	— 0.46
„	„	28.	51.5	55.7	60	29.776	84 18 63	528.18	— 0.27
„	„	31.	47.4	51.2	55	29.627	84 18 63	528.50	— 1.22
„	Aug.	2.	56.1	57.1	59.8	29.762	84 18 72	523.14	+ 1.12
„	„	3.	52.1	56	59.2	30.026	84 18 57	532.28	+ 0.99
„	„	4.	52.1	57.8	59.2	30.184	84 18 62	529.28	— 4.75
„	„	11.	51	56.2	58.2	30.169	84 18 61	530.05	— 5.02
„	„	12.	49.1	56	58	30.124	84 18 49	537.07	+ 0.74
„	„	24.	54.8	57.7	60	29.745	84 18 81	518.55	— 4.74
„	„	26.	51	54	59.1	29.620	84 18 68	526.98	+ 1.70
„	Sept.	5.	54.9	57	60.8	29.442	84 18 85	517.95	+ 0.15
„	„	11.	50.8	53.7	58	29.735	84 18 76	522.60	— 5.14

$23 \times d\text{R} = -32''.34$ $\qquad d\text{R} = -1''.41$

$\kappa = 9.8637$ $\qquad d\mu = -0''.142$

Fomalhaut.

$^* \delta = -30° \; 31'. \; 15''.26.$ (H. J.)

Precession $= +19''.073$; sec var. $= +0''.135$; proper motion $= -0''.180$.

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1839,	Oct.	12.	44.8	46.8	48.9	29.710	84° 39′. 95	566.45	+ 2.07
„	„	17.	39.1	44.9	46.5	29.944	84 39 82	575.00	— 1.11
„	„	27.	41.1	45	47	30.293	84 39 70	583.29	+ 3.22
„	„	28.	43.1	46.2	48.5	30.412	84 39 94	569.18	— 10.86

* Airy, (Greenwich, 22 obs.) .	16″.00	Johnson, ...	14″.75
„ (Cambridge, 21) . . .	13 .38	Mine, ...	14 .35
Henderson, (Cape,) . . .	15 .78	Bessel, (Tab. Reg.)	20 .24

DATE.			E. T.	I. T.	A. T.	BAROM.	ZEN. DIST.	OBS. REFRACT.	dR.
1839,	Nov.	11.	42.9	44	47.3	28.998	84° 40′. 18	555.57	+ 2.57
„	„	12.	40.9	43	46.1	29.332	84 40 17	557.07	— 4.92
„	„	26.	32	35.8	40	29.173	84 39 83	578.74	+ 8.02
„	Dec.	2.	38.2	40.8	42.4	29.758	84 40 00	568.91	— 5.24
„	„	28.	29.8	34.2	37.2	29.762	84 39 82	579.90	— 5.53
1840,	Sept.	28.	47.1	50	51.1	29.016	84 39 83	550.25	+ 2.36
„	„	29.	45.1	47.8	49.1	29.582	84 39 54	568.21	+ 2.97
„	Oct.	2.	42	45	46.1	30.148	84 39 46	574.50	— 1.28
„	„	3.	39.5	47	49.8	30.160	84 39 45	574.35	— 4.67
„	„	4.	40.8	46	46.8	30.119	84 39 47	572.97	— 3.79
„	„	10.	41.8	43.8	45.5	30.210	84 39 42	577.23	+ 0.09
„	„	11.	42.8	45.2	46	30.295	84 39 42	577.20	— 0.26
„	„	12.	45.9	47.5	49	30.405	84 39 52	571.35	— 4.20
„	„	14.	41.2	43.2	45.5	30.208	84 39 31	584.18	+ 6.84
„	Nov.	21.	43.9	42.8	43.8	29.470	84 39 84	556.80	— 4.16
„	„	27.	41.4	42.8	43	30.130	84 39 70	565.65	— 11.08

$20 \times d\text{R} = -28''.96$ $\qquad d\text{R} = -1''.45$

$\kappa = 10.6207$ $\qquad d\mu = -0.136$

Combining, we obtain,

NAME.	NO. OBS.	$nd\text{R} \times \kappa$	$n\kappa^2$.	$d\mu$.
o^2 Canis.	13	— 112.9960	354.772	— 0″.318
15 Argûs.	16	— 80.9305	490.286	— 0 165
o^1 Canis.	13	— 130.2914	400.634	— 0 325
ξ Argus.	13	— 126.6950	436.280	— 0 290
λ Sagittarii.	23	+ 48.3397	856.812	+ 0 056
Antares.	22	— 157.9369	878.733	— 0 180
δ Canis Maj.	14	— 193.3463	608.381	— 0 318
σ Sagittarii.	17	— 112.9772	778.032	— 0 145
ϵ Canis Maj.	16	— 162.8188	1193.730	— 0 136
η Canis Maj.	16	— 162.7435	1255.767	— 0 130
δ Sagittarii.	18	— 396.3260	1648.873	— 0 241
ζ Canis.	17	— 337.5532	1722.926	— 0 196
ζ Sagittarii.	23	— 318.9921	2237.730	— 0 142
Fomalhaut.	20	— 307.5758	2255.988	— 0 136
Sum . .	241	— 2552.8420	15118.944	

$$\text{and } d\mu = \frac{-2552.842}{15118.944} = -0.1688$$

The correction for run for these stars give,

$$d^2\mu = \frac{-95.5541}{15118.944} = -0.0063$$

and we have,

$$\begin{array}{r} \mu = 57.7682 \\ -\ \ 0.1688 \\ -\ \ 0.0063 \\ \hline 57.5931 \end{array}$$

which agrees so nearly with the determination from sub-polar stars (their difference being only $0''.5$ at Fomalhaut) that there is obviously no necessity for supposing any discrepancy between the northern and southern refractions at this observatory, especially as it would vanish entirely were the Cape declinations not used. If now we take $\mu = 57.546$; the value of $\frac{l}{a}$ reduced to my latitude is 0.00129263, and (using the well-known notation of Mr. Babbage to save space) the equation of refraction becomes for $\text{T} = 50$, barometer 29.60,

$$\text{R} = \text{tang} \,.\, \theta \times \log^{-1}(1.7600151)$$

$$+ \text{tang}^3 \,.\, \theta \times \log^{-1}(7.9045751)\,\{1 + \text{tang}^2 \,.\, \theta \times \log.^{-1}(6.44559)\}$$

$$- \frac{\text{tang}}{\cos^2} \,.\, \theta \times \log^{-1}(8.8715498)\,\{1 + \text{tang}^2 \,.\, \theta \times \log.^{-1}(6.77484)\}$$

$$+ \frac{\text{tang}^3}{\cos^2} \,.\, \theta \times \log^{-1}(6.3720995)\,\{1 + \text{tang}^2 \,.\, \theta \times \log^{-1}(7.06014)\}$$

$$- \frac{\text{tang}^5}{\cos^2} \,.\, \theta \times \log^{-1}(4.0315728)\,\{1 + \text{tang}^2 \,.\, \theta \times \log^{-1}(7.23971)\}$$

$$+ \frac{\text{tang}^7}{\cos^2} \,.\, \theta \times \log^{-1}(1.7907405)\,\{1 + \text{tang}^2 \,.\, \theta \times \log^{-1}(7.34007)\}$$

From this the following tables have been computed. In the first, the column A contains the logarithm of $\frac{\mu(1+\epsilon(\text{T}-50))}{29.60}$, and B that of $\frac{1+\epsilon'(\text{T}-50)}{1+\epsilon''(\text{T}-50)}$, ϵ' the expansion of the brass scale being taken $= 0.000010479$; and ϵ'' that of mercury $= 0.0001$.

The second table contains C, the sum of all the terms except the first, for the argument zen. distance; D = the change of C for one degree increase of temperature; and E its change for one inch rise of the barometer. This last serves also to change C for a slight variation in μ, the constant, for

$$\frac{d\text{C}}{d\mu} = \text{E} \times 0.5144$$

and A must be changed by $\log \mu' - \log \mu$.

The refraction is given by

$$\log \text{R}' = \text{A} + \text{B} + \log \text{tang apparent zen. dist.} + \log. \text{bar.}$$
$$\text{R} = \text{R}' - \text{C} - \text{D} \times (\text{T} - 50°) - \text{E} \times (\text{bar.} - 29.60.)$$

Argument of A, external thermometer = T

Argument of B, attached thermometer = τ

Argument of C, D, and E, apparent zenith distance.

TABLE I.

Ther. = 50° ; bar. = 29.60 inches.

T	A.	B.	T.	A.	B.	T.	A.	B.
0	0.33343$_{94}$		31	0.30517$_{88}$	+ 74	62	0.27864$_{83}$	− 46
1	0.33249$_{94}$		32	0.30429$_{88}$	+ 70	63	0.27781$_{83}$	− 50
2	0.33155$_{94}$		33	0.30341$_{88}$	+ 66	64	0.27698$_{82}$	− 54
3	0.33061$_{94}$		34	0.30253$_{88}$	+ 62	65	0.27616$_{82}$	− 58
4	0.32968$_{94}$		35	0.30165$_{87}$	+ 58	66	0.27534$_{83}$	62
5	0.32874$_{93}$		36	0.30078$_{87}$	+ 54	67	0.27451$_{82}$	− 66
6	0.32781$_{93}$		37	0.29991$_{87}$	+ 50	68	0·27369$_{82}$	− 70
7	0.32688$_{93}$		38	0.29904$_{87}$	+ 46	69	0.27287$_{82}$	− 74
8	0.32595$_{92}$		39	0.29817$_{87}$	+ 42	70	0.27205$_{82}$	− 78
9	0.32503$_{92}$		40	0.29730$_{87}$	+ 39	71	0.27123$_{81}$	− 81
10	0.32411$_{92}$		41	0.29643$_{86}$	+ 35	72	0.27042$_{81}$	− 85
11	0.32319$_{92}$		42	0.29557$_{86}$	+ 31	73	0.26961$_{81}$	− 89
12	0.32227$_{92}$		43	0.29471$_{86}$	+ 27	74	0.26880$_{81}$	− 93
13	0.32135$_{91}$		44	0.29385$_{87}$	+ 23	75	0.26799$_{81}$	− 97
14	0.32044$_{91}$		45	0.29298$_{86}$	+ 19	76	0.26718$_{81}$	− 101
15	0.31953$_{91}$		46	0.29212$_{86}$	+ 15	77	0.26637$_{80}$	− 105
16	0.31862$_{91}$		47	0.29126$_{85}$	+ 11	78	0.26557$_{81}$	− 109
17	0.31771$_{91}$		48	0.29041$_{85}$	+ 7	79	0.26476$_{80}$	− 113
18	0.31680$_{91}$		49	0.28956$_{84}$	+ 3	80	0.26396$_{80}$	− 117
19	0.31589$_{90}$		50	0.28872$_{85}$	0	81	0.26316$_{80}$	− 121
20	0.31499$_{90}$	+ 117	51	0.28787$_{84}$	− 3	82	0.26236$_{80}$	− 125
21	0.31409$_{90}$	+ 113	52	0.28703$_{85}$	− 7	83	0.26156$_{80}$	− 129
22	0.31319$_{89}$	+ 109	53	0.28618$_{84}$	− 11	84	0.26076$_{80}$	− 132
23	0.31230$_{90}$	+ 105	54	0.28534$_{85}$	− 15	85	0.25996$_{80}$	− 136
24	0.31140$_{89}$	+ 101	55	0.28449$_{84}$	− 19	86	0.25916$_{79}$	− 140
25	0.31051$_{90}$	+ 97	56	0.28365$_{84}$	− 23	87	0.25837$_{79}$	− 144
26	0.30961$_{89}$	+ 93	57	0.28281$_{84}$	− 27	88	0.25758$_{79}$	− 148
27	0.30872$_{89}$	+ 89	58	0.28197$_{84}$	− 31	89	0.25679$_{79}$	− 152
28	0.30783$_{89}$	+ 85	59	0.28113$_{83}$	− 35	90	0.25600$_{79}$	− 156
29	0.30694$_{88}$	+ 81	60	0.28030$_{83}$	− 39	91	0.25521$_{79}$	− 160
30	0.30606$_{89}$	+ 78	61	0.27947$_{83}$	− 42	92	0.25442	− 163

TABLE II.

Z. D.	C.	D.	E.	Z. D.	C.	D.	E.	Z. D.	C.	D.	E.
4°	0.01			76° 20′	4.69_{35}	0.002	0.14	81° 55′	20.82_{60}	0.005	0.63_{1}
10	0.01_{1}			40	5.04_{38}	0.002	0.15	82 0	21.42_{63}	0.005	0.64_{2}
15	0.02_{1}			77 0	5.42_{42}	0.002	0.16	5	22.05_{65}	0.005	0.66_{2}
20	0.03_{1}			20	5.84_{47}	0.002	0.18	10	22.70_{68}	0.006	0.68_{2}
25	0.04_{1}			40	6.31_{52}	0.002	0.19	15	23.38_{70}	0.006	0.70_{3}
30	0.05_{2}			78 0	6.83_{28}	0.002	0.21	20	24.08_{73}	0.006	0.73_{2}
35	0.07_{3}			10	7.11_{30}	0.002	0.21	25	24.81_{76}	0.006	0.75_{2}
40	0.10_{5}			20	7.41_{31}	0.002	0.22	30	25.57_{78}	0.006	0.77_{2}
45	0.15_{1}			30	7.72_{33}	0.002	0.23	35	26.35_{83}	0.006	0.79_{3}
46	0.16_{1}			40	8.05_{35}	0.003	0.24	40	27.18_{85}	0.007	0.82_{3}
47	0.17_{1}			50	8.40_{36}	0.003	0.25	45	28.03_{89}	0.007	0.85_{2}
48	0.18_{1}			79 0	8.76_{39}	0.003	0.26	50	28.92_{93}	0.007	0.87_{3}
49	0.19_{1}			10	9.15_{42}	0.003	0.28	55	29.85_{97}	0.007	0.90_{3}
50	0.20_{1}		0.01	20	9.57_{44}	0.003	0.29	83 0	$30.82_{1.00}$	0.008	0.93_{3}
51	0.21_{2}		0.01	30	10.01_{46}	0.003	0.30	5	$31.82_{1.06}$	0.008	0.96_{3}
52	0.23_{2}		0.01	40	10.47_{49}	0.003	0.31	10	$32.88_{1.10}$	0.008	0.99_{4}
53	0.25_{2}		0.01	50	10.96_{53}	0.003	0.33	15	$33.98_{1.15}$	0.009	1.03_{3}
54	0.27_{2}		0.01	80 0	11.49_{28}	0.003	0.35	20	$35.13_{1.19}$	0.009	1.06_{4}
55	0.29_{3}		0.01	5	11.77_{28}	0.003	0.35	25	$36.32_{1.24}$	0.010	1.10_{4}
56	0.32_{3}		0.01	10	12.05_{29}	0.003	0.36	30	$37.56_{1.31}$	0.010	1.14_{4}
57	0.35_{4}		0.01	15	12.34_{30}	0.004	0.37	35	$38.87_{1.37}$	0.011	1.18_{4}
58	0.39_{4}		0.01	20	12.64_{31}	0.004	0.38	40	$40.24_{1.42}$	0.012	1.22_{5}
59	0.43_{4}		0.01	25	12.95_{33}	0.004	0.39	45	$41.66_{1.50}$	0.013	1.27_{4}
60	0.47_{5}		0.01	30	13.28_{33}	0.004	0.40	50	$43.16_{1.57}$	0.013	1.31_{5}
61	0.52_{6}		0.02	35	13.61_{35}	0.004	0.41	55	$44.73_{1.64}$	0.014	1.36_{5}
62	0.58_{7}		0.02	40	13.96_{35}	0.004	0.42	84 0	$46.37_{1.72}$	0.015	1.41_{6}
63	0.65_{7}		0.02	45	14.31_{36}	0.004	0.43	5	$48.09_{1.80}$	0.016	1.47_{6}
64	0.72_{8}		0.02	50	14.67_{38}	0.004	0.44	10	$49.89_{1.89}$	0.018	1.53_{6}
65	0.80_{11}		0.03	55	15.05_{40}	0.004	0.45	15	$51.78_{1.99}$	0.019	1.59_{6}
66	0.91_{12}		0.03	81 0	15.45_{41}	0.004	0.46_{2}	20	$53.77_{2.09}$	0.022	1.65_{7}
67	1.03_{14}		0.03	5	15.86_{42}	0.004	0.48_{1}	25	$55.86_{2.20}$	0.023_{2}	1.72_{7}
68	1.17_{17}		0.04	10	16.28_{44}	0.004	0.49_{1}	30	$58.06_{2.31}$	0.025_{3}	1.79_{8}
69	1.34_{21}	0.000	0.04	15	16.72_{45}	0.004	0.50_{2}	35	$60.37_{2.45}$	0.028_{3}	1.87_{8}
70	1.55_{25}	0.001	0.05	20	17.17_{47}	0.004	0.52_{1}	40	$62.82_{2.58}$	0.031_{4}	1.95_{9}
71	1.80_{29}	0.001	0.06	25	17.64_{48}	0.005	0.53_{1}	45	$65.40_{2.71}$	0.035_{4}	2.04_{9}
72	2.09_{39}	0.001	0.06	30	18.12_{50}	0.005	0.54_{2}	50	$68.11_{2.89}$	0.039_{5}	2.13_{10}
73	2.48_{49}	0.001	0.08	35	18.62_{52}	0.005	0.56_{2}	55	$71.00_{3.06}$	0.044_{6}	2.23_{11}
74	2.97_{62}	0.001	0.09	40	19.14_{54}	0.005	0.58_{1}	85 0	74.06	0.050	2.34
75	3.59_{78}	0.001	0.11	45	19.68_{56}	0.005	0.59_{1}				
76	4.37_{32}	0.001	0.13	50	20.24_{58}	0.005	0.60_{3}				

Example.

Fomalhaut, zen. dist. 84° 39′. 46; E. T. 42°; bar. 30^{i}.148; A. T. 46°.1.

tang Z. D. .	1.02913		C $-$ 62.56
A. .	0.29557		(D) $+$ 0.25 $= -8^{7} \times -0.031$
B. $+$	15		(E) $-$ 1.01 $= +0.548 \times -1.95$
30.148	1.47926		$-$ 63.38
	2.80411	. . . .	636.96 = R′
			573.58 = R.

The Reader is requested to make the following Correction:—

Page 223, last line, *for* 1 + *read* 1 —.